TEACHING NOTE 01-02:
INTRODUCTION TO INTEREST RATE OPTIONS

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Most of the time when people talk about options, they are talking about options on stocks. It may be surprising to find out that the options on interest rates may well be much more widely used. Options on interest rates are very similar to ordinary stock options, but there are many important differences.

First, let us start off with a reminder of what an option is: *a contract between two parties that permits one party, the option buyer, to buy or sell an underlying asset at a fixed price*. The counterparty is the option seller or writer. The option buyer pays a premium up front and obtains the right to buy or sell the asset for a specific period of time, which ends at the option expiration day. If the option is used to buy or sell the underlying asset, it is said to be exercised. An option granting the right to buy is referred to as a call; an option granting the right to sell is referred to as a put. An option that can be exercised any time prior to expiration is referred to as an American option. An option that can be exercised only at expiration is referred to as a European option.

With interest rate options, the underlying is not an asset. It is an interest rate. The underlying should actually be thought of as an interest payment. Therefore, we define an interest rate call option as an option that grants the holder the right to make a fixed or known interest payment and receive a variable or unknown interest payment. An interest rate put option is an option that grants the holder the right to make a variable or unknown interest payment and receive a fixed or known interest payment.

Interest rate options are, by definition, cash settled. The underlying is an interest payment. When the option is exercised, the short makes a net payment to the long. The payment is adjusted by three factors. One is that, as always, interest rates are quoted as annual rates, but the rate itself might be for a period of less than a year. Thus, an adjustment such as multiplying by days/360 or days/365 is done. Also, to compute an interest payment requires the multiplication of a rate times an amount on which the rate is applied. For loans, this amount is commonly referred to as the loan principal or balance of the loan. Accordingly, an interest rate option has a designated principal, called the notional principal, on which the interest payment is calculated.

Finally, we should note a very important characteristic of interest rate options. In the Eurodollar market, when a rate is determined, it is the rate on a Eurodollar time deposit determined in London. It refers to an instrument on which the interest accrues and is paid at a later date. For example, suppose a 90-day Eurodollar time deposit is created. The rate is 5% and the deposit is in the amount of $1 million. When the deposit matures in 90 days, the amount it pays off is

$$\$1,000,000(1 + 0.05(90/360)) = \$1,012,500.$$

In other words, the interest is added on to the principal. So when an interest rate is determined in the LIBOR market, it is assumed that the interest will be paid a certain number of days later.

The interest rate options market uses the LIBOR market as the basis for its underlying rate.[1] The parties agree that the payoff will be determined by the official LIBOR rate.[2] When LIBOR is determined in the Eurodollar time deposit market, it is assumed that the interest is paid a certain number of days later. The interest rate options market follows a similar rule. Thus, when an interest rate option based on m-day LIBOR expires, the actual payoff to the holder of the call occurs m days later.

This practice may seem unusual and disadvantageous, but in fact, it is precisely what most buyers of interest rate options prefer. The holder of an interest rate call option in all likelihood is using it to offset the risk associated with a floating rate loan. If a company is borrowing at a floating rate, the rate will be set on a given day, the interest will accrue for a certain number of days, and then the interest will be paid.

Because, as we shall cover later, interest rate options are most often used to hedge interest rates that will be set on specific known future dates, they tend to be European style.

Examples of Interest Rate Options

Let us take a look at how all of this works with an example. Consider a call option in which the underlying is the rate on 90-day LIBOR. We shall say that the option expires in 30 days. The buyer of the option designates an exercise price, which is in the

[1]Some interest rate options are based on other rates, but LIBOR is the most common rate used.

[2]The official LIBOR rate is published by the British Bankers Association (BBA) and is based on a sampling of the LIBORs quoted by a designated group of London Banks. The BBA obtains these quotes, drops a certain number of the highest and lowest quotes and averages the remaining quotes to obtain its official designated LIBOR.

form of a rate. In fact, it is better to call this the exercise rate, but sometimes it is called the strike rate or strike. Suppose the exercise rate is 6%. The notional principal will be $10 million. This option grants the holder the right to make a 6% interest payment on $10 million and receive an interest payment to be determined by the rate on 90-day LIBOR in 30 days. For this right, the buyer pays a premium. Let us first focus on how the payoff is determined.

Suppose it is now 30 days later. The table below indicates the payoffs of the option based on a range of possible values of LIBOR at expiration of the option.

LIBOR	Payoff
0.04	$0
0.05	$0
0.06	$0
0.07	$25,000
0.08	$50,000

In general, the payoff of an interest rate call is determined by the formula

$$(\text{Notional Principal})\text{Max}(0, \text{LIBOR} - \text{Strike})\left(\frac{\text{days}}{360}\right).$$

So in this case, the payoff will be

$$(\$10,000,000)\text{Max}(0,\text{LIBOR} - 0.06)\left(\frac{90}{360}\right).$$

So in all cases where LIBOR is below 6%, the payoff is zero. In the case where LIBOR is 7%, the payoff is

$$(\$10,000,000)\text{Max}(0,0.07 - 0.06)\left(\frac{90}{360}\right) = \$25,000.$$

Interest rate put options grant the right to receive the strike rate and pay LIBOR. In general the payoff of an interest rate put is

$$(\text{Notional Principal})\text{Max}(0, \text{Strike} - \text{LIBOR})\left(\frac{\text{days}}{360}\right).$$

Thus, an interest rate put will pay off when LIBOR at the expiration of the option is below the strike rate.[3]

The option premium is usually quoted in terms of basis points. For example, a premium might be 20 basis points. The actual cash amount of the premium is then computed as the quoted premium times the notional principal times days/360, or whatever day count adjustment is being used.

Applications of Interest Rate Options

Interest rate options permit the buyer of the option to pay a premium up front and obtain protection against either rising or falling interest rates. Consider for example, a company that plans to borrow $10 million in 30 days at 90-day LIBOR plus 200 basis points. Thus, in 30 days, the firm will take out the loan at the 90-day LIBOR on that day plus 200 basis points. Then 90 days later, the firm will pay back the principal along with the interest. The firm is exposed to the risk associated with LIBOR in 30 days. Even though it is concerned about higher interest rates, it would also like to benefit from falling interest rates. Thus, it is willing to pay a premium to obtain protection from rising rates while enabling it to benefit from falling rates. Let us say it selects a strike of 6% and pays a premium of 50 basis points. That means it will pay

$$\$10,000,000(0.005)\left(\frac{90}{360}\right) = \$12,500$$

as the option premium up front.

In 30 days the firm takes out the loan at whatever LIBOR is that day plus 200 basis points. Thus, 90 days later it will owe

$$\$10,000,000\left(1 + (LIBOR + 0.02)\left(\frac{90}{360}\right)\right).$$

The option expires when the loan is taken out and pays off

$$\$10,000,000 Max(0, LIBOR - 0.06)\left(\frac{90}{360}\right)$$

90 days later. Thus, 90 days after taking out the loan, the firm will pay the former amount less the latter, which will be

[3]Note in all cases that the use of the formula days/360 is not universal. In some cases, the parties agree to use 365 days. Also, *days* can be based on 90, 180, 360, etc. or can involve the actual number of days between two designated dates.

$$\$10,000,000\left(1+(Min(LIBOR,0.06)+0.02)\left(\frac{90}{360}\right)\right).$$

In other words, the firm will pay 2% plus either LIBOR or 6%, whichever is smaller.

Of course, we have not taken into account the fact that the firm had to pay a premium of $12,500 up front. Let us say that the rate the firm could earn on if it invested the $12,500 would be 5.5%. So if it pays $12,500 up front to buy the option, it is equivalent to paying $12,500(1 + 0.055(30/360)) = $12,557 at the time the loan is taken out. This increases the cost of the loan in that the firm receives only $10 million minus the equivalent of $12,557.

Firms that lend money at a floating rate are concerned about the possibility of falling rates. Therefore, a firm that plans to make a loan at a future date would consider the possibility of buying an interest rate put. A put would pay off when LIBOR at expiration is below the strike rate. The payoff formula would be

$$(\text{Notional Principal})\text{Max}(0,\text{Strike}-\text{LIBOR})\left(\frac{days}{360}\right).$$

The firm would have to pay the option premium up front, which reduces the effective rate on the loan, but would be compensated when LIBOR falls below the strike rate. It would benefit when LIBOR rises above the strike rate, as the option expires with no value but the loan rate is higher.

Finally, we should note that if the firm did not want to pay cash up front, it would have to be willing to set the rate, thereby giving up the right to benefit from falling or rising rates. In this situation, it would use an FRA, which is a forward contract where the underlying is an interest rate.

Interest Rate Caps, Floors, and Collars

In the examples we have seen so far, there was but a single interest rate to be set. This type of situation is a loan in which the rate is fixed only once during the life of the loan. Many types of loans involve multiple rate resets. For example, firms often take out floating rate loans, in which each 90 or 180 days the rate is reset, interest then accrues, and then the interest is paid, at which time a new rate is set. The principal is paid back at the maturity date of the loan.

For each loan reset date a borrower or lender firm is exposed to interest rate risk Therefore, to manage that risk the firm might choose to purchase options expiring on each of the reset dates. Because of the heavy demand for these combinations of options, derivatives dealers offer them in the form of packages of instruments referred to as caps and floors. A cap is a combination of interest rate call options. A floor is a combination of interest rate put options. The component options are called *caplets* and *floorlets*. Each component option is independent of the other options. That is, whether one option is exercised is not affected by whether another option is exercised. Each option is designed to compensate the borrower or lender for the exposure to the underlying rate on the designated rate reset date. The payoffs are calculated exactly as we have already shown.

Consider a firm that takes out a two-year loan of $20 million. The loan rate is set every six months. Interest then accrues for six months and is paid. Then the rate is reset. The underlying rate will be 180-day LIBOR plus 100 basis points.[4] On the day the firm takes out the loan, the rate is set for the first six months. There is no way to hedge that rate.[5] The firm is exposed to the rate reset in six months, twelve months, and 18 months. Let us say it purchases a cap consisting of component caplets that expire on the dates on which the rates will be reset. An exercise rate of 5% is selected, and the notional principal is set at the face amount of the loan of $20 million. Let L_0 be LIBOR on the day the loan is taken out.

In 180 days, the firm will owe

$$\$20,000,000(L_0 + 0.01)\left(\frac{180}{360}\right).$$

At this time, we observe a new value for 180-day LIBOR, which is denoted as L_{180}. This means that the new interest rate will be $L_{180} + 0.01$. At this time, however, a caplet is expiring and will pay

$$\$20,000,000 Max(0, L_{180} - 0.05)\left(\frac{180}{360}\right).$$

Thus, 180 days later when the interest is paid, the amount the firm will owe is

[4]180-day LIBOR is the rate that London banks borrow in dollars for 180 days from other London banks.

[5]Of course, when the firm first anticipated taking out the loan, it was exposed to the first setting of the rate. It might well have hedged that rate with an interest rate call option. In this example, we shall assume that the firm is currently at the date on which the loan is taken out. Thus, it is too late to hedge the initial setting of the rate.

$$\$20,000,000(Min(L_{180},0.05)+0.01)\left(\frac{180}{360}\right).$$

In other words, it will owe 1% at a minimum and either 5% or LIBOR, whichever is lower. This continues for two more rate resets. In each case, the firm will owe interest at 1% plus either 5% or LIBOR on the rate reset date, whichever is lower.

An interest rate floor works in a similar manner, but the payoff is that of an interest rate put, as covered earlier. Of course, interest rate floors are used by lenders to protect against falling rates, while leaving the opportunity to benefit from rising rates.

For both caps and floors, a premium must be paid up front. The premium will be the sum of the prices of the component caplets or floorlets.

Buyers of caps are often concerned about paying cash up front to receive the protection they desire. There are two ways they can avoid the payment of cash up front. One is to use a swap. While a swap locks in the fixed rate, it does not allow the opportunity for the borrower to benefit from falling rates. An alternative strategy is called a *collar*, the purchase of a cap and sale of a floor. The premium received from the sale of the floor offsets the premium paid for the purchase of the cap. In constructing a strategy containing the purchase of a cap and sale of a floor, there are a number of considerations.

If the cap and floor have the same exercise rates, then the collar will be similar to a swap. Consider below the simple case of a $1.00 notional principal with an annual rate, the latter of which means that days/360 is just 360/360. This reduces our notation somewhat.

The payoff of the cap is

Max(0,LIBOR – Strike)

Recalling that we are the seller of the floor, its payoff is

-Max(0,Strike – LIBOR).

Thus, the combined payoff will be

LIBOR – Strike.

Since the borrower is paying LIBOR on its loan, the net effect is that the borrower will pay the strike rate. This effectively converts the loan to a fixed rate loan and would seem to be like a swap, which converts a floating rate loan to a fixed rate loan. But a swap has a unique fixed rate that results in the present value of the fixed payments equaling the present value of the floating payments. If any other fixed rate is used in a swap, the two streams of payments are not equal. This would mean that the swap has a non-zero value up front and would require a payment from one party to the other. Thus, the combination of cap and floor could also require a fixed payment up front, meaning that the floor premium could be more or less than the cap premium.

If, however, the strike rate on the floor were equal to the fixed rate on the swap, then the cap and floor premiums would offset. In other words, a long cap and short floor with strike rates equal to the fixed rate on a swap would have offsetting premiums, and, therefore, cost nothing up front. But the problem with this strategy is that it would no longer have the option-like characteristics that allow the benefits of a favorable rate movement while protecting against an unfavorable rate movement.

A collar effectively takes care of this problem by separating the exercise prices of the cap and floor. Consider a borrower who buys a cap with a strike of 7%. This party pays a premium up front and is protected against increases in interest rates above 7%. Suppose the party sells a floor with an exercise rate of 5%. The counterparty, who is the buyer of the floor, receives the benefit of decreases in interest rates below 5%. Thus, the borrower will have to pay when interest rates are below 5%. For the willingness to pay when rates are below 5%, the borrower receives a premium. This premium can be used to partially or fully offset the premium on the cap.

For a given exercise rate on the cap, there is a unique exercise rate on the floor that will produce a premium received on the floor that offsets the premium paid on the cap. In this note, we do not get into how interest rate option premiums are determined, but in general an option pricing model would have to be used. Trial and error with different exercise prices would be used to determine the exercise rate on the floor that produces a floor premium that offsets the cap premium.

Of course it is not necessary that the cap and floor premiums offset. If the floor premium is less than the cap premium, the buyer of the collar would have to pay some amount of money up front. If the floor premium exceeds the cap premium, the buyer of the collar would actually receive some money up front.

Let us assume that the 5% is the exercise rate on the floor that generates the same premium on the floor as the premium on the cap with 7% exercise rate. This means that the borrower is subject to some uncertainty when LIBOR is in the range of 5% to 7%. If the borrower wants to cap the rate at 6%, the cap would be more expensive. This means that he would need to make the floor more expensive. To make a put more expensive, one must raise the exercise rate. Suppose the floor rate must be raised to 5.75%. This means that the borrower will have to give up the benefit of interest rate decreases below 5.75%, which is more costly than giving up interest rate decreases below 5%. Of course, there is no single best strategy for setting the exercise rates. The borrower would have to combine its expectations about interest rates along with its tolerance for risk.

There are some exotic varieties of caps and floors, but most take the general form described here.

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