To understand how to determine the values of forward contracts, options, and swaps, we need to examine some basic principles that comprise the foundation for valuing assets, after which we will extend those principles to valuing derivatives. The procedures for valuing forwards, options, and swaps differ somewhat but rest upon a few basic ideas. But first we look at valuing a risky asset.

**Valuing Risky Assets**

Suppose there is an asset that will have a value of either $100 if a good economy occurs or $50 if a bad economy occurs one period later. The interest rate for that period is 5%. The probability of the good economy is 60%, and the probability of the bad economy is 40%. What should be the current price of the asset?

First we have to start by finding the expected value of the asset one period later:

\[E = 100(0.6) + 50(0.4) = 80\]

An investor who is indifferent to risk, who we refer to as being *risk neutral*, would value the asset at

\[\frac{80}{1.05} = 76.19\]

If he made this investment many times over, he would receive an average payoff of $80. If the investor paid $76.19 each time, he would effectively break even, being paid only a 5% return to compensate him for the time value of money. Risk neutral investors do not demand compensation for the risk. They are perfectly content to make $x one time and lose $x another time.

A risk averse investor would not be satisfied to make an investment many times and earn an average return that compensates him only for the time value of money. If he made an investment that earned $x one time and lost $x another time, he would wish he had not made the investment at all. In this case, he would pay less than $76.19. How much less? That depends on how he feels about risk. The more risk averse he is, the less
he would pay. Thus, the more risk averse he is, the more he will discount the expected value to obtain the price he will pay. People vary in their degrees of risk aversion. Some are highly risk averse and some only moderately risk averse. Let us assume that the investor believes a price of $70 would be appropriate. What does this imply about his risk aversion?

At a price of $70, he has an expected return of

$$\frac{80}{70} - 1 = 0.1429$$

Thus, his expected return is 14.29%. If he makes this investment many times, he will earn on average 14.29%. Given a 5% risk-free rate, he will earn a risk premium of 9.29%. Thus, he is pricing the investment such that he expects a 9.29% risk premium. Of course, he may make the investment only one time, in which case he will either earn $100 - $70 = $30 or lose $70 - $50 = $20. Or he might make the investment several times and make or lose an average of some value other than the expected value of $76.19. But his long-run expectation is to earn a return of 14.29%, which will compensate him for taking the risk.

**Identical Assets and the Law of One Price**

Suppose there is another asset trading in a different market. If a good economy occurs, the asset will be worth $200, and if a bad one occurs, it will be worth $100. Notice that this second asset pays exactly two times the payoff of the good asset. Suppose that the first asset is trading for the price of $76.19, and the second asset is trading for $160. This situation creates an unstable condition that will bring out a different class of traders called *arbitrageurs* who trade to earn risk-profits without committing any of their own money.¹

These arbitrageurs will execute a very simple strategy. They will sell short the second asset, thereby collecting $160 while promising to buy that asset back at $200 if the good economy occurs or $100 if the bad one occurs.² Now, they have $160. They

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¹Some arbitrageurs take little if any risk, but there are also investors who ordinarily take risks who will notice the opportunity and engage in an arbitrage transaction.

²Short selling involves borrowing the asset from someone else, selling it to another party, and committing to repurchase the asset at a later date to return it to the original owner.
take $152.38 and buy two shares of the first asset. They keep the remaining $160 - $152.38 = $7.62. \(^3\) Let us see what happens for both possible outcomes.

**A good economy occurs:**

They are holding two shares of the first asset, which are collectively worth $200. They take the $200 and buy one share of the second asset, which they then deliver to cover their short position.

**A bad economy occurs:**

They are holding two shares of the first asset, which are collectively worth $100. They take the $100 and buy one share of the second asset, which they then deliver to cover their short position.

In both outcomes, the shares they own will be precisely paid to repurchase the shares they have sold short. Thus, they are immune to the risk. They do not care which outcome occurs, because in either case they simply end up with the $7.62 and interest on it. They have, in effect, manufactured money without taking any risk. This type of transaction is called *arbitrage*.

Naturally many investors will rush to do this transaction, which will put downward pressure on the price of the second asset or upward pressure on the price of the first asset, or a combination of both. Very quickly the two asset prices must align themselves so that the second asset price is twice that of the first. Under the principle of arbitrage, prices are in equilibrium relative to each other. We do not know or care if the first asset is priced correctly or the second asset is priced correctly. We simply care whether the two prices properly aligned, in this case such that the second asset is worth twice the price of the first.

This result illustrates the principle called the *Law of One Price*: Identical assets cannot sell for different prices. These may not appear to be identical assets, but their returns are perfectly synchronized. Regardless of which outcome occurs, the second asset’s value is twice that of the first. Of course, you might wonder what would happen if investors in the second market had a different opinion of the price of the second asset. Given their perhaps lower degree of risk aversion, they might find a price of $80 to be perfectly fine. Then, how could arbitrageurs make their profits off of their pricing the

\(^3\)That money would be invested to earn the risk-free rate of interest.
asset seemingly too high? This situation could not remain because the arbitrageurs would drive the prices together. The two sets of investors could not disagree about the price. If they did disagree, we would not need arbitrageurs to reconcile their differences. They would simply trade with each other. If one thought it was worth $80 and the other thought it was worth $70, they each improve their positions by trading with each other in a price in between. In fact, the existence of investors with different opinions about what an asset is worth is the basis for most trading in asset markets.

**Derivative Contracts**

Suppose we create a side bet on the outcome of the first asset. Suppose Trader A offers to pay Trader B $26.50 if the good economy occurs, provided that Trader B will pay Trader A $23.50 if the bad economy occurs. Suppose that Trader A borrows $70 at the risk-free rate of 5% and uses the funds to purchase the asset. He then enters into this side bet to pay Trader B $26.50 if the good economy occurs with the condition that he receive $23.50 from Trader B if the bad economy occurs. The $70 he borrows will have to be paid back, thereby requiring a payment of $70(1.05) = $73.50 one period later. His overall payoff will be

*A good economy occurs:*

He holds an asset worth $100, owes $26.50, and pays back the loan value of $73.50 for a net of zero.

*A bad economy occurs:*

He holds an asset worth $50, receives $23.50, and pays back the loan value of $73.50 for a net of zero.

Of course, there is no particular reason this investor should do this. He committed none of his own funds at the start and ends up with no money regardless of the outcome. But this example illustrates that the side bet should pay $26.50 to one party if a good economy occurs and $23.50 to the other party if the bad economy occurs. These are fair, equilibrium terms for the side bet.

What if the side bet pays $26.50 if the good economy occurs and $25 if the bad one occurs? Then it should be apparent that if the bad economy occurs, Trader A will end up with $50 + $25 - $73.50 = $1.50. If the good economy occurs, he will end up with zero. So, think about that. Without investing of his own money, Trader A takes a
gamble in which he either ends up with no money or $1.50. He will certainly do it, as well as will other arbitrageurs. Their collective actions force the terms of the side bet to change until the bet is a fair one for both parties. That, naturally, occurs if the payoff in the bad economy is $23.50.

The two payoffs to Trader A, $26.50 in the good economy and -$23.50 in the bad economy, can be derived as follows:

*If the good economy occurs:*

\[ 100 - x = 26.50 \]

*If the bad economy occurs:*

\[ 50 - x = -23.50 \]

Clearly \( x = 73.50 \). This number is a special value and is the key to finding the equilibrium payoffs of the side bet. We could, however, have obtained that value in another and much simpler way. We simply take the price of the asset, $70, and compound it at the risk-free rate, 5%.

\[ 70(1.05) = 73.50 \]

This side bet illustrates one type of derivative transaction known as a *forward contract*. Embedded in a forward contract is the notion of a *forward price*. We can think of the forward price, $73.50 in this example, as the price that one party agrees to pay for the asset when the contract expires. Thus, here a party going long the forward contract agrees to pay $73.50 one period later. If the good economy occurs, the party pays $73.50 and receives the asset worth $100 for a net gain of $26.50. If the bad economy occurs, the party pays $73.50 and receives the asset worth only $50 for a net loss of $23.50. Thus, a forward contract is an agreement for one party to pay the forward price and receive the asset at a later date. The other party agrees to deliver the asset and receive the forward price.

In this example, we assumed that the asset price was $70, which was the price appropriate for the highly risk averse investor. In the example we looked at concerning the valuation of risky assets, we had an investor who was less risk averse and considered the asset worth $80. In that case, the forward price would be \( 80(1.05) = 84 \). The forward contract would need to pay $16 to one party if the good economy occurs and $34 to the other party if the bad economy occurs. In this way, if someone did the transaction
described just above, that is, borrowed $80 at 5% and offered the forward contract as a seller, he would end up with $100 - $16 - $84 = $0 if the good economy occurs and $50 + $34 - $84 = $0 if the bad economy occurs. This is as it should be. The person committed no personal funds and took no risk. Therefore, the gain should be zero for certain.

The point is that the forward price, whether $70 or $84, depends on the spot price of the underlying asset and the risk-free rate. It will also depend on how long the contract is and for some assets and some derivatives, it will depend on other factors. The most important point is that we take the asset price as given. We do not care whether the investors who priced the asset were highly risk averse, slightly risk averse, or risk neutral. We assume that how investors feel about risk is fully incorporated into the price of the asset. Once the asset price is determined, we do not care what factors investor considered in obtaining that price.

With that in mind, we can treat our process of valuing derivatives as though investors are risk neutral. Consider the case where investors are highly risk averse and price the asset at $70. We noted that at a risk-free rate of 5%, these investors are pricing the asset with a risk premium of 9.29%, because the expected value of the asset is $80, which is 14.29% higher than $70, but a portion of the 14.29% is the risk-free return (5%). For our forward contract, however, the forward price was found to be $73.50, which is only 5% higher than the spot price of $70. Thus, the forward contract can be viewed as being priced by risk neutral investors. That does not mean that investors are actually risk neutral. In fact, we just said they are highly risk averse in this example. But the forward price is found by taking the spot price of $70 and factoring it up to reflect only a risk-free return of 5%, thereby obtaining $73.50 as the forward price. The principle that derivative instruments are priced to prohibit the opportunity to earn an arbitrage profit turns out to be the same as the notion that we can treat participants in derivative markets as though they are risk neutral. This simply means that derivatives are priced as though investors are risk neutral. In fact, they are not risk neutral but are risk averse. Their risk aversion is embedded into the price of the underlying asset. Given the price of the underlying asset, it is easy to price the derivative, using, of course, the assumption of risk neutrality.

\[4\]For forward contracts on some assets, it will depend on costs of carrying or storing those assets and any payments, such as dividends, that those assets make. For options, it depends on all of these factors as well as the volatility of the underlying asset.
Yet another way to look at it is that if we did not treat investors as though they were risk neutral when pricing derivatives, we would be double counting the risk. The risk is incorporated into the price of the asset. The price of the asset is then incorporated into the derivative price. This takes care of the risk. We cannot treat the derivatives investor as though he requires a premium as compensation for the risk. That premium is already there - in the price of the asset.