TEACHING NOTE 96-01:
DEFAULT RISK AS AN OPTION

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Option pricing theory provides a rich framework for analyzing many types of financial transactions. One particularly fruitful approach is to view the stock as an option on the assets of the firm. This provides important insights into the nature of the claims of stockholders and bondholders and more clearly shows the role of default risk as the option held by the stockholders and written by the creditors. Although implementing this model in practice can be quite difficult, it is nonetheless widely used in the analysis of credit risk. One well-known company uses it as the basis of its commercially successfully credit risk service. The model is oftentimes known as the *Merton model*, for Robert C. Merton who advanced its theoretical development with a 1974 paper. It should be noted, however, that the basic idea was presented in the classic Black and Scholes article of 1973 that launched their model.

Consider a firm with assets currently worth a market value of A. The firm has debt with a face value of F, due in T years, and equity with a market value of S. The market value of the debt is B. By definition, $A = B + S$. Let the continuously compounded risk-free rate be r and the volatility of the assets be σ.

When the debt is due, the assets will be worth A_T. Since A_T can be less than F, default is possible. Consider the following outcomes:

If $A_T > F$, the bondholders receive F and the stockholders receive the residual value, $A_T - F$

If $A_T \le F$, the bondholders receive A_T and the stockholders receive nothing.

These results are summarized in the following table:

Suppliers of Capital	Payoffs to Suppliers of Capital at Bond Maturity	
	If $A_T \leq F$	If $A_T > F$
Bondholders receive	A_T	F
Stockholders receive	0	$A_T - F$
Total capital distributed	A_T	A_T

The position of the stockholders is equivalent to that of an investor holding a call option but where the underlying asset is not the stock, but rather, the assets of the firm. The stockholders have the implicit right to exercise their option, which they do if the assets are worth more than they owe on the debt. In that case, they pay the face value of the debt, which is like the exercise price, and claim the residual value. If the face value of the debt is worth more than the value of the assets, the stockholders default, thereby allowing their option to expire worthless.

Recall that put-call parity for a European option on a non-dividend paying stock is

$$C = P + S - Xe^{-rT}$$

where C is the call price, P is the put price, S is the underlying stock price, X is the exercise price, r is the risk-free rate and T is the time to expiration of the option. Making the analogy, we can express the stock as a call option on an underlying asset, which is actually the assets of the firm:

$$S = P + A - Fe^{-rT}.$$

This decomposition shows that a share of stock is equivalent to a put option, a long position in the assets and a risk-free bond with face value of F. The latter two terms can be viewed as the leverage inherent in a stock. We can also express the market value of the bonds in terms of put-call parity. Since by definition, the market value of the debt is the market value of the assets minus the market value of the stock (B = A – S), we simply substitute the right-hand side of the previous equation for S, giving us

$$\begin{aligned} B &= A - S \\ &= A - (P + A - Fe^{-rT}) \\ &= -P + Fe^{-rT}. \end{aligned}$$

This decomposition shows that a bond that is subject to default is equivalent to a risk-free bond with a value of Fe^{-rT} and a short put with a value of -P. This means that the bondholders in effect hold a risk-free bond with risk added by selling the stockholders a put.

The stockholders, therefore, own a put option, which reflects their limited liability. It is their right to discharge fully their debt to the bondholders by simply turning over the assets to the bondholders. They will do this if the value of the assets is worth less than the face value of the debt when the debt matures. What lawyers view as the principle of limited liability, financial economists more rightly view as a put option.

The Black-Scholes model provides additional insights into the nature of the stockholder and bondholder positions. Recall the Black-Scholes formula for an ordinary call on a stock:

$$C = SN(d_1) - Xe^{-rT}N(d_2)$$

where $d_1 = (\ln(S/X) + (r + \sigma^2/2)T)/\sigma\sqrt{T}$ and $d_2 = d_1 - \sigma\sqrt{T}$. The analogous expression for the stock as a call option on the assets is

$$S = AN(d_1) - Fe^{-rT}N(d_2)$$

where $d_1 = (\ln(A/F) + (r + \sigma^2/2)T)/\sigma\sqrt{T}$ and d_2 is defined as before. Remember that σ is the volatility of the underlying asset, which is the firm=s assets in this case. Using the expression B = A – S, we now substitute the above definition for S and obtain

$$B = A[1 - N(d_1)] + Fe^{-rT}N(d_2).$$

This decomposition of the market value of the debt shows that there are two components, $A[1 - N(d_1)]$, and $Fe^{-rT}N(d_2)$. In the risk neutral framework in which the Black-Scholes model can be derived, $N(d_2)$ is the probability that the option is exercised. $N(d_1)$, however, is not easily interpreted as a probability by itself. Jointly with A, however, the expression can be interpreted as the discounted expected value of the assets at expiration, conditional on the asset value exceeding F, times the probability that the asset value will exceed F. With the expression written as $1 - N(d_1)$, we can interpret $A[1 - N(d_1)]$ as the discounted expected value of the assets, conditional on the asset value being less than F, times the probability that the asset value is less than F. The second term in the above expression, $Fe^{-rT}N(d_2)$, is the present value of the payoff of the bond times the probability the bond will be paid off, or in other words, the expected payoff of the bond under risk neutrality.

Thus, the creditors have a risky position that will pay off possibly the bond's face value and if not, then the value of the assets. Although debt is rarely viewed that way, it is clear that the bondholders have granted a valuable option to the stockholders, which is the right of the stockholders to discharge fully their obligation by turning over the assets to the bondholders.

Note also that the model allows for yield spreads to vary according to the quality of the debt. Define $B = Fe^{-kT}$ as the market value of the debt, which is the discounted face value. The rate k is the discount rate, which consists of the risk-free rate and a risk premium to reflect the default risk.[1] The difference between k and r can be interpreted as the yield spread between a default-free bond and one with a positive probability of default. It can be shown that k will vary with the volatility of the assets, the amount of debt a company has and when that debt is due. The yield spread will generally be greater the more volatile the assets and the greater the debt load relative to the assets. Whether the yield spread is greater with respect to maturity depends on other factors as well. A firm deeply in debt with debt due soon is likely to default so the yield spread is higher. A firm deeply in debt with the debt due much later can have a lower yield spread. It benefits from having a longer time to straighten out its problems, which it may have a good chance to do if the volatility is high.

While the model in this form applies only to a firm with a single zero coupon bond issue, it has been extended to cases of multiple issues and coupon-bearing bonds. Indeed such complex capital structures characterize most firms. Practitioners make a number of simplifying assumptions in order to implement the model. In many cases, academics would be uncomfortable with these assumptions, but the models are, nonetheless, widely used.

References

The original idea for the stock as an option on the assets was suggested in the celebrated Black-Scholes article,

Black, F. and M. Scholes. The Pricing of Options and Corporate Liabilities. *Journal of Political Economy* 81 (May-June, 1973), 637-659.

The actual development of the idea is in

[1]With a constant risk-free rate, there is no interest rate risk so any risk premium must reflect only default risk.

Merton, R. C. On the Pricing of Corporate Debt: The Risk Structure of Interest Rates. *The Journal of Finance* 29 (May, 1974), pp. 449-470.

with a minor adjustment by

Lee. C. J. The Pricing of Corporate Debt: A Note. *The Journal of Finance* 36 (December, 1981), 1187-1189.

The extension to multiple and subordinated debt issues is in

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The extension to coupon bonds is in

Geske, R. The Valuation of Corporate Liabilities as Compound Options. *Journal of Financial and Quantitative Analysis* 12 (November, 1977), 541-552.

The following paper uses the option pricing to model to illustrate many principles of corporate financial theory:

Galai, D. and R. W. Masulis. The Option Pricing Model and the Risk Factor of Stock. *Journal of Financial Economics* 3 (January-March, 1976), 53-81.