

TEACHING NOTE 97-08:
PRICING AND VALUATION OF COMMODITY SWAPS

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The commodity swap is designed to assist in the management of the risk associated with the prices of input resources such as energy, precious metals, and agricultural products. To date, however, commodity swaps are predominantly associated with energy-related products. The transaction involves the exchange of payments between two parties at a series of scheduled future dates. At least one set of payments is determined by the price of a commodity. The other set is usually a fixed rate. For example, suppose an airline has determined that it must purchase 500,000 barrels of jet fuel per quarter. It enters into a swap with a dealer in which the airline pays the dealer a fixed rate for 500,000 barrels of jet fuel on a specific date each quarter. The dealer in turn pays the airline the price of jet fuel for 500,000 barrels on those same dates. In this manner, the airline has fixed the purchase price of its fuel for the term of the swap. Typically the swap is settled in cash, meaning that no physical delivery of the commodity occurs.

A fairly common procedure used in commodity swaps is to have the variable payment set at the average price of the commodity over a specific period, rather than the price of the commodity on the settlement date. This type of arrangement makes commodity swaps similar in principle to the derivatives known as *Asian options*, wherein the payoffs are determined by the average price of the asset during a period over the option's life. Asian-style payoffs accomplish two objectives. They structure the payoff to reflect only the average performance of the asset, which can be useful to firms whose risks are associated more with the average value of the asset. For example, a firm may know that it needs to purchase about 100,000 units of a commodity over a six-month period. Its purchases will be made weekly, but it does not know exactly which amounts will be purchased from week to week. It knows only that its total purchases are likely to add up to around 100,000 units. It may, therefore, feel that its exposure is more to the average price of the asset during the six-month period. An Asian-style swap could better

suit its needs than one that pays off based on the final price of the commodity at the end of the six-month period.

Asian-style settlement is also useful in markets where the commodity is quite volatile, and particularly when its price is susceptible to distortion or even manipulation. Oil is a good example of such a market. Extreme events such as tension in the middle east can lead to sharp increases in the price of oil. In addition, the oil market is characterized by a fairly small number of producers, many of which belong to OPEC. Consequently, oil prices can possibly be manipulated by collusive practices of suppliers.

In this Teaching Note, we shall look at how commodity swaps are priced and valued. Remember that pricing refers to the process of determining the fixed rate that must be paid at each settlement date in exchange for receiving a payoff tied to the price of the asset. Valuation means to determine what the swap is worth later during its life. For presentation purposes we shall refer to swaps that pay off the asset price as standard swaps and swaps that pay off the average asset price as Asian swaps, although as noted above, the Asian swap is more common.

We begin by establishing a time framework. Let there be discrete time points denoted as $t = 0, 1, 2, \dots, T$ that are the swap settlement dates. The asset prices on these dates are $S(1), S(2), \dots, S(T)$. For standard swaps the price is denoted as $G(0,T)$, which is read as the price of a standard swap started at time 0 and ending at time T with settlements at the discrete equally-spaced time points 1, 2, ..., T. From the perspective of the party paying the fixed rate and receiving the asset price, who is called the “long,” the payoffs are $S(1) - G(0,T), S(2) - G(0,T), \dots, S(T) - G(0,T)$. Now consider a time j , where $0 < j < T$, at which the asset price is $S(j)$. We will want to find the value of the swap at that time. The risk-free rate per period is r and the asset’s yield per period is δ , which is essentially the asset’s convenience yield less storage costs. If there are no storage costs, then $\delta > 0$. Both r and δ are assumed to be non-stochastic.¹ Thus, from time t to $t+1$ the net cost of carry factor is $e^{(r-\delta)}$. When we price the Asian swap, we shall introduce some additional notations.

¹The assumption the interest rate is non-stochastic is necessary for pricing the Asian swap but not for the standard swap.

Pricing the Standard Swap

We wish to determine the value $G(0,T)$, which is defined as the fixed payment such that the present value of the fixed payments equals the present value of the variable payments, implying an initial value of zero. The present value of the fixed payments is simply

$$G(0, T) \left(\frac{1 - e^{-rT}}{e^r - 1} \right).$$

The present value of the variable payments can be determined by purchasing units of the asset that will yield these cash inflows. Since the asset accrues value at the rate of δ per period, we should purchase less than one unit of the asset to produce one unit at a future date. In other words, at time 0 we purchase $e^{-\delta t}$ units of the asset for all $t = 1, 2, \dots, T$. Then at each time $t = 1, 2, \dots, T$ we sell one unit of the asset at the price $S(t)$. This will generate the necessary cash inflow. The number of units of the asset that we must purchase at time 0 is

$$X(0, T) = \sum_{t=1}^T e^{-\delta t} = \frac{1 - e^{-\delta T}}{e^\delta - 1}.$$

The total cash outlay at time 0 will be $X(0,T)S(0)$. The value of the swap at time 0 must be zero since no money changes hands, so we equate the cash outlay required to reproduce the variable cash inflows to the present value of the fixed cash outflows:

$$X(0, T)S(0) = G(0, T) \left(\frac{1 - e^{-rT}}{e^r - 1} \right),$$

and solving for $G(0,T)$ gives

$$G(0, T) = X(0, T)S(0) \left(\frac{e^r - 1}{1 - e^{-rT}} \right).$$

The appendix contains a numerical example.

Valuation of the Standard Swap

Now suppose we are positioned during the life of the swap. Let k represent the number of the previous settlement date, where k can be $0, 1, 2, \dots, T-1$.² Then let j be a fraction, $0 < j < 1$, representing the portion of the current settlement that has passed since

²In the case where $k = 0$, we are in the first settlement period and there is no previous settlement.

time k . Define $R = T - k$ as the number of remaining settlements. We position ourselves at time kj .³ We wish to find $V(kj;0,T)$ defined as the value of the swap at time j when the last settlement was at time k , the swap started at time 0 and ends at time T . If we are long this swap, the remaining cash flows are $S(k+1) - G(0,T)$, $S(k+2) - G(0,T)$, ..., $S(T) - G(0,T)$. We can now short a swap with the same remaining settlement dates. This new swap will have no value or cash flow at time kj but will have the cash flows $G(kj,T) - S(k+1)$, $G(kj,T) - S(k+2)$, ..., $G(kj,T) - S(T)$ where $G(kj,T)$ is the price of the new swap. The combined cash flows from our two positions are now $G(kj,T) - G(0,T)$ at each time $t = k+1, \dots, T$. The value of this combined position is obviously the present value of this stream of cash flows, which, because they are certain, can be obtained by discounting at the risk-free rate. The value of this position is the value of the original swap minus the value of the new swap, which equals the value of the original swap. Defining PVFA as the present value factor at kj for the remaining cash flows, the price of the new swap is, using the results of the previous section, $G(kj,T) = X(kj,T)S(kj)/PVFA$ where $X(kj,T)$ is the number of units of the asset that must be purchased at time kj to reproduce the variable swap payments on the new swap. Then the value of the old swap is

$$\begin{aligned} V(kj;0,T) &= [G(kj,T) - G(0,T)]PVFA \\ &= [X(kj,T)S(kj)/PVFA - G(0,T)]PVFA \\ &= X(kj,T)S(kj) - G(0,T)PVFA. \end{aligned}$$

The interpretation of this formula is that the value of the swap to pay $G(0,T)$ and receive the asset price at each settlement is the value of a position of $X(kj,T)$ units of the asset worth $S(kj)$ and a liability of payments of $G(0,T)$ at each remaining settlement date. To value the swap then requires that we determine $X(kj,T)$ and PVFA. This general interpretation will apply to each case for any kind of commodity swap.

To find the number of units to purchase at time kj , we step back to time k , the previous settlement. At time k the number of units to purchase would be

$$\frac{1 - e^{-\delta R}}{e^{\delta} - 1}.$$

³Time kj is not the product of k time j .
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At time kj , this would be found by compounding the above factor at the rate δ for the period j to obtain

$$X(kj, T) = \left(\frac{1 - e^{-\delta R}}{e^\delta - 1} \right) e^{\delta j}.$$

The present value factor for an annuity at time k would be

$$\frac{1 - e^{-rR}}{e^r - 1}.$$

At time kj this would be found by compounding at the rate r for the period j to obtain

$$\left(\frac{1 - e^{-\delta R}}{e^\delta - 1} \right) e^{rj}.$$

Thus, we have the value of the swap as

$$V(kj; 0, T) = X(kj, T)S(kj) - G(0, T) \left(\frac{1 - e^{-rR}}{e^r - 1} \right) e^{rj}.$$

Asian Swap Pricing

Now we wish to determine the price of the Asian swap, which we denote as $G_A(0, T)$. This is the fixed rate that would equate the present value of the cash inflows to the present value of the cash outflows. The present value of the outflows is

$$G_A(0, T) \left(\frac{1 - e^{-rT}}{e^r - 1} \right).$$

Each inflow is defined as the average of N prices observed on specific dates during the settlement interval. The settlement inflows are denoted as $A(1), A(2), \dots, A(T)$. We define each settlement interval as consisting of times $n = 1, 2, \dots, N$, which we call price observation dates. A price, denoted as $S(tn)$ where t is the upcoming settlement, is observed and recorded at time n . The average at t is

$$A(t) = \sum_{n=1}^N S((t-1)n) / N.$$

Each price observed in recording the average is denoted as $S((t-1)1), S((t-1)2), \dots$, which will appear as $S(01), S(02), \dots$ or $S(11), S(12), \dots$.

To reproduce a cash inflow at time t equivalent to the average price as defined above, we must undertake at time 0 a set of transactions as follows:

To produce the value $A(1)$ at time 1, we

buy $e^{-(\delta/N)}(1/N)e^{-(r/N)(N-1)}$ units. At time 01, this will accrue to $[e^{-(\delta/N)}(1/N)e^{-(r/N)(N-1)}]e^{(\delta/N)} = (1/N)e^{-(r/N)(N-1)}$ units. We sell the units at price $S(01)$ and invest the proceeds in riskless bonds maturing at time 1. At time 0 The accrued value at time 1 will then be $(1/N)e^{-(r/N)(N-1)}S(01)e^{(r/N)(N-1)} = (1/N)S(01)$. We also buy $e^{-(\delta/N)2}(1/N)e^{-(r/N)(N-2)}$ units. At time 02, this will accrue to $[e^{-(\delta/N)2}(1/N)e^{-(r/N)(N-2)}]e^{(\delta/N)2} = (1/N)e^{-(r/N)(N-2)}$ units. We sell these units at the price $S(02)$ and invest the proceeds in riskless bonds maturing at time 1. The accrued value at time 1 will be $(1/N)e^{-(r/N)(N-2)}S(02)e^{(r/N)(N-2)} = (1/N)S(02)$. Additional transactions following this pattern are also conducted at time 0 so that at time 1, the accrued value will be $(1/N)(S(01) + S(02) + \dots + S(0N))$. The total number of units to purchase at time 0 to create this cash flow is

$$(1/N) \sum_{n=1}^N e^{-(\delta/N)n} e^{-(r/N)(N-n)}.$$

These transactions have reproduced only the first cash inflow, $A(1)$. To reproduce the second cash inflow, we do a set of similar transactions. In general we purchase enough units of the asset so that when the price observation date is reached, we sell off the appropriate number of units, invest the proceeds in riskless bonds so that at the next settlement the amount accrued will be $(1/N)$ times the price recorded for averaging. Doing this for all price observation dates reproduces the average at the next settlement date. Doing this for all settlement dates reproduces the swap's entire cash inflows.

The total number of units to be purchased at time 0 is

$$X_A(0, T) = (1/N) \sum_{t=1}^T \sum_{n=1}^N \left\{ e^{-(\delta/N)[(t-1)N+n]} e^{-(r/N)(N-n)} \right\}.$$

This expression can be simplified. The term in braces can be written as $e^{-(r-\delta)} e^{-\delta t} e^{n(r-\delta)/N}$. Putting this expression back into the summation gives

$$X_A(0, T) = (1/N) e^{-(r-\delta)} \left(\sum_{t=1}^T e^{-\delta t} \right) \left(\sum_{n=1}^N e^{n(r-\delta)/N} \right).$$

Using the mathematical rule that $\sum_{j=1}^N Y^j = Y(1 - Y^N)/(1 - Y)$, the above expression can be written as

$$X_A(0, T) = (1/N) \left(\frac{1 - e^{-\delta T}}{e^\delta - 1} \right) \left(\frac{e^{-(r-\delta)} - 1}{e^{-(r-\delta)/N} - 1} \right).$$

To price the swap we set its value at time 0 to zero. This is done by setting the value of the assets purchased to the value of the cash outflows:

$$X_A(0, T)S(0) = G_A(0, T) \left(\frac{1 - e^{-rT}}{e^r - 1} \right).$$

Solving for the swap price, we have

$$G_A(0, T) = X_A(0, T)S(T) \left(\frac{e^r - 1}{1 - e^{-rT}} \right).$$

Valuation of the Asian Swap

Now suppose the last settlement date was time k ($k = 0, 1, \dots, T$). We are currently at time k_i where i is the integer number representing the i^{th} price observation recorded for the next settlement. A total of i prices, which includes the current price, have been observed and recorded during the current settlement. There are $N - i$ prices remaining to be observed and recorded in the current settlement and $R = T - k$ settlements remaining. In this case we are on a price observation date. Later in this section we shall look at the case where we are not on a price observation date. We wish to find the value of the swap, which is denoted as $V_A(k_i; 0, T)$. Recall that in general the value of this swap is the number of units of the asset we would need to purchase today to replicate the variable payments times today's price minus the present value of the remaining fixed payments. To replicate the variable payments, we must determine the number of units of the asset to purchase today, but we will also have to purchase bonds in an amount sufficient to grow at the riskless rate to the value $(1/N)(S(k_1) + S(k_2) + \dots + S(k_i))$, which reflects the prices already observed and recorded in the current settlement period and which will be used, along with the future prices $S(k, i+1), \dots, S(k, N)$, in computing the average at the next settlement date.

The number of units of the asset to purchase at time k can be found by first determining the number of units to purchase at time k to reproduce all of the swap's cash inflows starting at time k and subtracting the number of units needed to reflect the price observation dates from time 1 to time i .⁴ In other words, we have already passed i price observation dates, so we do not need to purchase assets to produce $(1/N)$ times the sum of the prices already observed on those dates; we shall, however, have to purchase bonds in an amount to reflect these already-observed prices, as noted above. At time k the number of units to purchase is found by using the formula we previously obtained and treating it as though the swap starts at time k . This formula would be

$$X_A(k, T) = (1/N) \left(\frac{1 - e^{-\delta R}}{e^\delta - 1} \right) \left(\frac{e^{-(r-\delta)} - 1}{e^{-(r-\delta)/N} - 1} \right).$$

Now we need to determine how many of these units would have been purchased to produce at the next settlement the amount $(1/N)(S(k_1) + S(k_2) + \dots + S(k_i))$, which are the previously recorded prices. In deriving the above formula in an earlier section, we used a more general formula involving summations:

$$(1/N) e^{-(r-\delta)} \left(\sum_{t=1}^T e^{-\delta t} \right) \left(\sum_{n=1}^N e^{n(r-\delta)/N} \right).$$

We need a special case of this formula, specifically that t goes from 1 to 1, since we are looking only at the upcoming settlement, and that n goes from 1 to i . This gives us

$$\begin{aligned} & (1/N) e^{-(r-\delta)} e^{-\delta} \left(\sum_{n=1}^i e^{n(r-\delta)/N} \right) \\ &= (1/N) e^{-r} \sum_{n=1}^i e^{n(r-\delta)/N}. \end{aligned}$$

Using again the rule for summation of a finite series of the above form and after some further algebraic rearrangements, we have

$$(1/N) e^{-r} \left(\frac{1 - e^{i(r-\delta)/N}}{e^{-(r-\delta)/N} - 1} \right).$$

Now the total number of units necessary at time k is

⁴One further adjustment will be needed, which we shall get to shortly.
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$$(1/N) \left\{ \left(\frac{1 - e^{-\delta R}}{e^\delta - 1} \right) \left(\frac{e^{-(r-\delta)} - 1}{e^{-(r-\delta)/N} - 1} \right) - e^{-r} \left(\frac{1 - e^{i(r-\delta)/N}}{e^{-(r-\delta)/N} - 1} \right) \right\}.$$

This is still not quite what we need, which is the quantity required at time ki and not at time k . We can obtain the desired value, however, by simply compounding the above quantity by the factor $e^{(\delta/N)i}$. Thus,

$$X_A(ki, T) = (1/N) e^{(\delta/N)i} \left\{ \left(\frac{1 - e^{-\delta R}}{e^\delta - 1} \right) \left(\frac{e^{-(r-\delta)} - 1}{e^{-(r-\delta)/N} - 1} \right) - e^{-r} \left(\frac{1 - e^{i(r-\delta)/N}}{e^{-(r-\delta)/N} - 1} \right) \right\}.$$

Now we must deposit enough cash to produce $(1/N)(S(k1) + S(k2) + \dots + S(ki))$ at the upcoming settlement. This is easily obtained by depositing the amount

$$B_A(ki, T) = (1/N) e^{-(r/N)(N-i)} \left(\sum_{n=1}^i S(kn) \right).$$

This amount will grow by the interest factor $e^{(r/N)(N-i)}$ and produce the desired total at the upcoming settlement.

Thus, the purchase of $X_A(ki, T)$ units of the asset at price $S(ki)$ and the purchase of riskless pure discount bonds worth $B_A(ki, T)$ will produce the cash inflows over the remaining life of the swap. The present value of the cash outflows is simply the present value of the remaining payments of $G_A(0, T)$,

$$G_A(0, T) \left(\frac{1 - e^{-rR}}{e^r - 1} \right) e^{(r/N)i}.$$

Recall that the swap value is simply the value of the assets required to replicate the cash inflows minus the present value of the remaining cash outflows on the swap. Thus,

$$V_A(ki; 0, T) = X_A(ki, T) S(ki) + B_A(ki, T) - G_A(0, T) \left(\frac{1 - e^{-rR}}{e^r - 1} \right) e^{(r/N)i}.$$

Now suppose that we are between two of the price observation dates. Let k be the previous settlement date, $k = 0, 1, 2, \dots, T$, and i be the previous price observation date. Now position ourselves at time h which is a fraction of the period from i to $i + 1$, the next price observation date. Note that $0 < h < 1$ and represents the elapsed time in the period, not the remaining time. We first solve for the number of units we would need to

purchase at this time to produce the cash inflows on the swap. We denote this value as $X_A(kih, T)$. By definition,

$$X_A(kih, T) = X_A(ki, T)e^{(\delta/N)h}.$$

The amount of cash we must invest in riskless bonds is

$$B_A(kih, T) = (1/N)e^{-(r/N)(N-i-h)} \left(\sum_{n=1}^i S(ki) \right),$$

which will grow by the interest factor of $e^{(r/N)(N-i-h)}$. The investment required to produce the cash inflows is $S(kih)X_A(kih, T) + B_A(kih, T)$. The present value of the remaining outflows is

$$G_A(0, T)e^{(r/N)(i+h)} \left(\frac{1 - e^{-rR}}{e^r - 1} \right).$$

The value of the swap is, therefore,

$$V_A(kih; 0, T) = S(kih)X_A(kih, T) + B_A(kih, T) - G_A(0, T)e^{(r/N)(i+h)} \left(\frac{1 - e^{-rR}}{e^r - 1} \right).$$

Appendix

This appendix provides numerical examples for each of the cases developed in the above sections. We shall work with a swap with four settlement dates. The asset yield is 0.03, the risk-free rate is 0.09, and the current asset price is 5.2630. For an Asian swap there will be three price observation dates. Thus, $T = 4$, $\delta = 0.03$, $r = 0.09$, $S(0) = 5.2630$, and $N = 3$. Other inputs will be provided as needed.

Let us price the standard swap. We first compute $X(0, 4)$, the number of units we must purchase to produce the cash inflows of the swap, as

$$X(0, 4) = \frac{1 - e^{-0.3(4)}}{e^{0.3} - 1} = 3.7131.$$

Then the swap price is found as

$$G(0, 4) = 3.7131(5.2630) \left(\frac{e^{0.09} - 1}{1 - e^{-0.09(4)}} \right) = 6.0874.$$

Thus, paying 6.0874 at each of the four settlement dates is a fair exchange for receiving the price of the asset on those settlement dates. Note that the swap price is considerably

higher than the current asset price. This is because the cost of carry, $r - \delta$, is relatively high at $0.09 - 0.03 = 0.06$. Recall that in forward contracts the excess of the forward price over the spot is directly related to the cost of carry. Since a swap is just a combination of forward contracts, that relationship continues to hold.

Now let us move forward past the first settlement date and $1/3$ of the way into the second. The current price is now 5.2852. Let us find the new number of units that we would have to purchase to produce the remaining cash inflows:⁵

$$X(11/3, 4) = \left(\frac{1 - e^{-0.03(3)}}{e^{0.03} - 1} \right) e^{0.03(1/3)} = 2.8545.$$

Then the value of the swap is

$$V_A(11/3; 0, T) = 2.8545(5.2852) - 6.0874 \left(\frac{1 - e^{-0.09(3)}}{e^{0.09} - 1} \right) e^{0.09(1/3)} = -.6743.$$

Now let us price the Asian version of this swap. With $N = 3$ prices in the average, the number of units required at time 0 to replicate the cash inflows is

$$X_A(0, 4) = (1/3) \left(\frac{1 - e^{-0.03(4)}}{e^{0.03} - 1} \right) \left(\frac{e^{-(0.09-0.03)} - 1}{e^{-(0.09-0.03)/3} - 1} \right) = 3.6431.$$

Solving for the swap price, we obtain

$$G_A(0, T) = 3.6431(5.2630) \left(\frac{e^{0.09} - 1}{1 - e^{-0.09(4)}} \right) = 5.9677.$$

Now let us value the Asian swap during its life. We first value it on a price observation date, the first such date after the first settlement; thus, $k = 1, i = 1$. Using the appropriate formula, we have

$$X_A(11, 4) = (1/3) e^{0.01(1)} \left\{ \left(\frac{1 - e^{-0.03(3)}}{e^{0.03} - 1} \right) \left(\frac{e^{-0.06} - 1}{e^{-0.02} - 1} \right) - e^{-0.09} \left(\frac{1 - e^{0.02(1)}}{e^{-0.02} - 1} \right) \right\} = 2.4846.$$

We must invest enough cash in riskless bonds to reflect the prices already observed in the average. There is only one previous price so we invest

$$(1/3)(5.2852)e^{-0.03(3-1)} = 1.6591.$$

⁵The notation below in the expression $X(11/3, 4)$ is not $11/3$ or $1/3$ but simply indicates that k is 1 and j is $1/3$.

in riskless bonds expiring in two periods. This amount will earn interest and grow to a value of $(1/3)(5.2852) = 1.7617$ at the next settlement. The value of the swap is

$$V(11;0,4) = (2.4846)(5.2852) + 1.6591 - 5.9677 \left(\frac{1 - e^{-0.09(3)}}{e^{0.09} - 1} \right) e^{0.03(1)} = -0.6603.$$

Now suppose we are not on a price observation date. Let us be 4/10 of the way between price observation dates 1 and 2, where the last settlement was at time 1. Thus, $k = 1$, $i = 1$, and $h = 0.4$. The current price is $S(11.4) = 5.3648$.⁶ The number of units of the asset we must buy is

$$X_A(11.4,4) = X_A(11,4)e^{0.01(4)} = 2.4846e^{0.01(4)} = 2.4946,$$

which makes use of the fact that we computed $X_A(11,4)$ in the previous problem. We must invest enough money in riskless bonds to reflect the prices already observed in the current settlement period. This will be

$$(1/3)(5.2852)e^{-0.03(3-1-0.4)} = 1.6792.$$

The value of the swap is then,

$$V_A(11.4;0,4) = 5.3648(2.4946) + 1.6792 - 5.9677 \left(\frac{1 - e^{-0.03(3)}}{e^{0.03} - 1} \right) e^{0.03(1.4)} = -2.5268.$$

References

This teaching note borrows from

Chance, D. M. and D. R. Rich. "Asset Swaps with Asian-Style Payoffs." *The Journal of Derivatives* 3 (Spring, 1996), 64-77.

Chance, D. M. and D. R. Rich. "Price the Average." *Energy and Power Risk Management* 1 (December, 1996/January, 1997), 22-25.

Other work on commodity swaps, though not accounting for the averaging effect, is in

Das, S. "Forward March." *Risk* 6 (February, 1993), 41-49.

Jarrow, R. and S. Turnbull. *Derivative Securities*. Cincinnati: South-Western (1996), pp. 444-447.

A good article discussing the corporate use of commodity swaps is

⁶Again, the notation $S(11.4)$ does not refer to time 11.4 but rather time $h = 0.4$ after time $i = 1$ after time $k = 1$.
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Schap, Keith. "Jet Fuel Swaps Ground Risk." *Futures* 15 (February, 1993), 44-46.