TEACHING NOTE 97-12:
CALCULATING THE BLACK-SCHOLES VALUE

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The purpose of this teaching note is to illustrate the very simple procedure for obtaining an option value from the Black-Scholes formula. We are given the following terms that go into the formula

S	=	current price of the underlying asset
X	=	exercise price of the option
r	=	risk-free rate
T	=	time to expiration
σ	=	standard deviation of the underlying asset
δ	=	yield of the underlying asset

Let us examine each of those values in turn.

The asset price and exercise price should be obvious. The risk-free rate is technically defined as the continuously compounded return on the risk-free security whose maturity best matches that of the option. Typically we use the U. S. Treasury bill that matures the day before the option expiration. Taking the maturity value of the Treasury bill, we divide by the price and then annualize. For example, if the bill matures in 33 days, which should be the option=s expiration give or take a day or two, and the discount rate is 5 percent, it is well-known that the T-bill price per $100 face value is $100 - 5(33/360) = $99.54. The return is 100/99.54 - 1, which is 0.0046 or 0.46 percent. To annualize this number, we do the following: $(100/99.54)^{365/33} - 1 = 0.0523$ or 5.23 percent. Then we take ln(1.0523) = 0.051 or 5.1 percent. Note, however, that the option price is not very sensitive to the risk-free rate so slight variations in this input value are not that critical.

The time to expiration is determined by dividing the number of days to expiration by 365. We simply take the number of days between two dates. If today is October 20 and the option expires on November 22, there are 11 more days in October and 22 in November for a total of 33 days. Then the time to expiration is 33/365 = 0.0904.

The volatility is the one completely unobserved value that goes into the model. It is defined as the standard deviation of the continuously compounded return on the stock over the upcoming life of the option. There is considerable disagreement about the best way to estimate the volatility, which should not be surprising since it is the one variable that is not easy to obtain.

Finally we have the yield on the asset. For a stock this would correspond to the dividend yield. For a bond, this would be the current (coupon) yield. For physical assets, this would correspond to an implicit (convenience) yield less any storage costs. When the underlying is a foreign currency, the yield is the foreign risk-free interest rate.

Recall that the Black-Scholes formula is as follows:

$$c = Se^{-\delta T} N(d_1) - Xe^{-rT} N(d_2)$$
$$p = Xe^{-rT} [1 - N(d_2)] - Se^{-\delta T} [1 - N(d_1)]$$
$$\textit{where}$$
$$d_1 = \frac{\ln(Se^{-\delta T}/X) + (r + \sigma^2/2)T}{\sigma\sqrt{T}}$$
$$d_2 = d_1 - \sigma\sqrt{T}$$
$$N(d_1), N(d_2) = \textit{cumulative normal probabilities}$$

Let us illustrate the calculation of the formula with the following problem: Consider an option on a barrel of oil in which the oil price is $22 a barrel, the exercise price is $24, the risk-free rate is 6.7 percent, the volatility is 8 percent, the option expires in three years and the oil has a convenience yield net of storage costs of 1 percent.

First calculate d_1:

$$d_1 = \frac{\ln(22e^{-(0.01)3}/24) + (0.067 + 0.08^2/2)3}{0.08\sqrt{3}}$$
$$= 0.675.$$

Then calculate d_2:

$$d_2 = 0.675 - 0.08\sqrt{3} = 0.536.$$

Then we must determine the normal probability. Teaching Note 97-01 reviews the normal probability distribution and provides a table and an analytic approximation. Another easy way to compute the normal probability is to use Excel's function =normsdist(). Thus, =normsdist(0.675) = 0.750162 and =normsdist(0.536) = 0.704021. Then we can complete the calculation as follows:

$$c = 22e^{-0.01(3)}0.750162 - 24e^{-0.067(3)}0.704021 = 2.196.$$

$$p = 24e^{-0.067(3)}[1 - 0.704021] - 22e^{-0.01(3)}[1 - 0.750162] = 0.476.$$

A put would be calculated as follows:

In practice the most difficult part of the calculations is obtaining the proper inputs. In many practical situations, only a rough estimate is needed. Consequently accuracy in the risk-free rate and yield can sometimes be sacrificed. The most important variable is the volatility, so it is of paramount importance that the volatility input be as accurate as possible.

References

Further details are contained in the following books:

Chance, D. M. and R. Brooks. *An Introduction to Derivatives*, 7th ed. (Mason, OH: Thompson-Southwestern, 2007), Ch. 5.

Chriss, N. A. *Black-Scholes and Beyond* (Chicago: Irwin Professional Publishing, 1997), Ch. 4.

Hull, J. C. *Options, Futures, and Other Derivatives*, 4th ed. (Upper Saddle River, New Jersey, 2000), Ch. 11.

Ritchken, P. *Derivative Markets: Theory, Strategy, and Applications* (New York: HarperCollins, 1996), Ch. 9.