

TN99-01: SOLVING LINEAR EQUATIONS IN EXCEL

Version date: August 21, 2008

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Suppose you have the following system of n linear equations in n unknowns:

$$m_1s_{11} + m_2s_{12} + \dots + m_ns_{1n} = c_1$$

$$m_1s_{21} + m_2s_{22} + \dots + m_ns_{2n} = c_2$$

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$$m_1s_{n1} + m_2s_{n2} + \dots + m_ns_{nn} = c_n$$

The value of each s_{ij} , $i = 1, \dots, n$; $j = 1, \dots, n$ is known as is each c_i , $i = 1, \dots, n$. The coefficients m_i , $i = 1, \dots, n$ are the unknowns. Assuming the equations are solvable, there are a number of ways to determine the solution. The use of Excel's matrix operations is a very simple and convenient way to do so.

Re-write the system of equations as follows. The unknowns can be written as an $n \times 1$ column vector called \mathbf{M} :¹

$$\mathbf{M} = \begin{pmatrix} m_1 \\ m_2 \\ \cdot \\ \cdot \\ \cdot \\ m_n \end{pmatrix}.$$

The known values of the s 's can be written as the $n \times n$ matrix \mathbf{S} :

$$\mathbf{S} = \begin{pmatrix} s_{11} & s_{12} & \cdot & \cdot & \cdot & s_{1n} \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ s_{n1} & s_{n2} & \cdot & \cdot & \cdot & s_{nn} \end{pmatrix}.$$

¹It is customary to use bold letters to identify matrices.

The known $c=s$ can be expressed as the $n \times 1$ column vector \mathbf{C} :

$$\mathbf{C} = \begin{pmatrix} c_1 \\ c_2 \\ \cdot \\ \cdot \\ \cdot \\ c_n \end{pmatrix}.$$

We can, therefore, write this system of equations as²

$$\mathbf{SM} = \mathbf{C}$$

The solution vector \mathbf{M} can, thus, be obtained using the inverse of \mathbf{S} :³

$$\mathbf{M} = \mathbf{S}^{-1}\mathbf{C}$$

In Excel create an array of n rows and n columns containing the s values. Create another array of n rows and one column containing the c values. Now we need to create an array to hold the inverse of \mathbf{S} . Select a block of empty cells containing n rows and n columns.⁴ Type the following: `=minverse(`, ending at the open parenthesis.⁵ Then select the cells containing the s values. Then type the close parenthesis `)`. Then hold down the Shift and Ctrl keys while you hit Enter. The inverse of \mathbf{S} will be entered in the block of cells you selected. This is called an Array Formula.

Now we must multiply the array containing the inverse by the \mathbf{C} array. Select an empty block of cells with n rows and one column. Type the following: `=mmult(`, ending at the open

²Recall how matrix multiplication works. A matrix with p rows and q columns can be multiplied by any matrix with q rows and any number, let's say w , columns. The resulting matrix will have p rows and w columns. When multiplying a row and column, an element in a given row is multiplied by the corresponding element in the given column and the sum of the products of the elements of a given row and column make up the corresponding element of the given row and column of the matrix that represents their product.

³Recall that the inverse of a matrix \mathbf{X} is another matrix, \mathbf{X}^{-1} , such that $\mathbf{X}^{-1}\mathbf{X} = \mathbf{I}$ where \mathbf{I} is the identity matrix, a matrix with the same number of rows and columns of \mathbf{X} and containing ones in the diagonal and zero's in the off-diagonal elements. An inverse can exist only for a square matrix and not every square matrix has an inverse.

⁴In each case in which you are instructed to select some empty cells, you can either hold down the shift key and use the cursor or use the left mouse button. Release the shift key and cursor or mouse button before typing.

⁵Here I shall use italics to indicate typed values.

parenthesis. Then select the cells containing the inverse of **S**. Then type a comma. Then select the cells containing the c values. Then type the close parentheses). Hold down the Shift and Ctrl keys and hit Enter. The solution will be contained in the cells you selected.

Let's work a problem in Excel. Consider a one-period binomial option pricing world. The stock is currently at 100 and can go up to 125 or down to 80. Thus, the up factor, u , is 1.25, and the down factor, d , is 0.80. The continuously compounded risk-free rate is 6.77% such that $\exp(0.0677) = 1.07$, the one-period interest factor. If we solve for the price of a call with an exercise price of 100, we obtain $c = 14.02$. Now let us see how to replicate the call using m shares of stock and B dollars invested in risk-free bonds.

Our replicating combination of stock and bonds is currently worth $100m + B$. If it does indeed replicate the call, one period later it will be worth $125m + 1.07B$, which has to equal 25, or $80m + 1.07B$, which has to equal 0. Thus, our equations are as follows:

$$125m + 1.07B = 25$$

$$80m + 1.07B = 0$$

Thus, in cell B6 enter 125, in C6 enter 1.07, in B7 enter 80 and in C7 enter 1.07. This is the **S** array. In cell E6 enter 25 and in E7 enter 0. This is the **C** vector.

Select cells B10:C11. Type the following: `=minverse(`, ending at the open parenthesis. Select the cells containing the **s** values, B6:C7. Type the close parenthesis). Then hold down the Shift and Ctrl keys while you hit Enter. The inverse of **S** should appear in cells B10:C11 and should be (rounded to 6 digits)

$$\mathbf{S}^{-1} = \begin{vmatrix} 0.022222 & -0.022222 \\ -1.661475 & 2.596054 \end{vmatrix}.$$

Select cells E10:E11. Type the following: `=mmult(`, ending at the open parenthesis. Select cells B10:C11. Then type a comma. Select cells E6:E7. Then type the close parenthesis). Hold down the Shift and Ctrl keys while you hit Enter. The solution should appear in cells E10 and E11. It should be:

$$\mathbf{M} = \begin{vmatrix} 0.555556 \\ -41.536864 \end{vmatrix}.$$

Thus, if we hold 0.555556 shares of the stock and borrow 41.536864 at 7%, we shall obtain the same

payoffs as the call option. Let's make sure this gives us the current call value:

$$(0.55556)100 - 41.536864 = 14.02,$$

which is the same current call option value we obtained using the formula.