Suppose you have the following system of n linear equations in n unknowns:

\[
m_1s_{11} + m_2s_{12} + \ldots + m_ns_{1n} = c_1 \\
m_1s_{21} + m_2s_{22} + \ldots + m_ns_{2n} = c_2 \\
\vdots \hspace{1cm} \vdots \\
m_1s_{n1} + m_2s_{n2} + \ldots + m_ns_{nn} = c_n
\]

The value of each \( s_{ij} \), \( i = 1, \ldots, n; j = 1, \ldots, n \) is known as is each \( c_i \), \( i = 1, \ldots, n \). The coefficients \( m_i \), \( i = 1, \ldots, n \) are the unknowns. Assuming the equations are solvable, there are a number of ways to determine the solution. The use of Excel’s matrix operations is a very simple and convenient way to do so.

Re-write the system of equations as follows. The unknowns can be written as an nx1 column vector called \( \mathbf{M} \):

\[
\mathbf{M} = \begin{bmatrix}
m_1 \\
m_2 \\
\vdots \\
m_n
\end{bmatrix}
\]

The known values of the s’s can be written as the n x n matrix \( \mathbf{S} \):

\[
\mathbf{S} = \begin{bmatrix}
s_{11} & s_{12} & \cdots & s_{1n} \\
\vdots & \vdots & \ddots & \vdots \\
s_{n1} & s_{n2} & \cdots & s_{nn}
\end{bmatrix}
\]

\[\text{It is customary to use bold letters to identify matrices.}\]
The known \( c = s \) can be expressed as the \( n \times 1 \) column vector \( \mathbf{C} \):

\[
\mathbf{C} = \begin{bmatrix} c_1 \\ c_2 \\ \vdots \\ c_n \end{bmatrix}
\]

We can, therefore, write this system of equations as\(^2\)

\[
\mathbf{S}\mathbf{M} = \mathbf{C}
\]

The solution vector \( \mathbf{M} \) can, thus, be obtained using the inverse of \( \mathbf{S} \):\(^3\)

\[
\mathbf{M} = \mathbf{S}^{-1}\mathbf{C}
\]

In Excel create an array of \( n \) rows and \( n \) columns containing the \( s \) values. Create another array of \( n \) rows and one column containing the \( c \) values. Now we need to create an array to hold the inverse of \( \mathbf{S} \). Select a block of empty cells containing \( n \) rows and \( n \) columns.\(^4\) Type the following:

\[
=\text{minverse}(, \text{ ending at the open parenthesis})\]

Then select the cells containing the \( s \) values. Then type the close parenthesis \( ) \). Then hold down the Shift and Ctrl keys while you hit Enter. The inverse of \( \mathbf{S} \) will be entered in the block of cells you selected. This is called an Array Formula.

Now we must multiply the array containing the inverse by the \( \mathbf{C} \) array. Select an empty block of cells with \( n \) rows and one column. Type the following: \( =\text{mmult}(, \text{ ending at the open parenthesis})\).
parenthesis. Then select the cells containing the inverse of \( S \). Then type a comma. Then select the cells containing the \( c \) values. Then type the close parentheses \( ) \). Hold down the Shift and Ctrl keys and hit Enter. The solution will be contained in the cells you selected.

Let's work a problem in Excel. Consider a one-period binomial option pricing world. The stock is currently at 100 and can go up to 125 or down to 80. Thus, the up factor, \( u \), is 1.25, and the down factor, \( d \), is 0.80. The continuously compounded risk-free rate is 6.77% such that \( \exp(0.0677) = 1.07 \), the one-period interest factor. If we solve for the price of a call with an exercise price of 100, we obtain \( c = 14.02 \). Now let us see how to replicate the call using \( m \) shares of stock and \( B \) dollars invested in risk-free bonds.

Our replicating combination of stock and bonds is currently worth \( 100m + B \). If it does indeed replicate the call, one period later it will be worth \( 125m + 1.07B \), which has to equal 25, or \( 80m + 1.07B \), which has to equal 0. Thus, our equations are as follows:

\[
\begin{align*}
125m + 1.07B &= 25 \\
80m + 1.07B &= 0
\end{align*}
\]

Thus, in cell B6 enter 125, in C6 enter 1.07, in B7 enter 80 and in C7 enter 1.07. This is the \( S \) array. In cell E6 enter 25 and in E7 enter 0. This is the \( C \) vector.

Select cells B10:C11. Type the following: \( = \text{minverse}() \), ending at the open parenthesis. Select the cells containing the \( s \) values, B6:C7. Type the close parenthesis \( ) \). Then hold down the Shift and Ctrl keys while you hit Enter. The inverse of \( S \) should appear in cells B10:C11 and should be (rounded to 6 digits)

\[
S^{-1} = \begin{bmatrix}
0.022222 & -0.022222 \\
-1.661475 & 2.596054
\end{bmatrix}.
\]

Select cells E10:E11. Type the following: \( = \text{mmult}() \), ending at the open parenthesis. Select cells B10:C11. Then type a comma. Select cells E6:E7. Then type the close parenthesis \( ) \). Hold down the Shift and Ctrl keys while you hit Enter. The solution should appear in cells E10 and E11. It should be:

\[
M = \begin{bmatrix}
0.555556 \\
-41.536864
\end{bmatrix}.
\]

Thus, if we hold 0.555556 shares of the stock and borrow 41.536864 at 7%, we shall obtain the same
payoffs as the call option. Let’s make sure this gives us the current call value:

\[(0.55556)100 - 41.536864 = 14.02,\]

which is the same current call option value we obtained using the formula.