The Alpha Bias in Asset Allocation Performance Measurement

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Professional asset allocation strategies frequently report positive alphas. This paper demonstrates that two-fund asset allocation strategies contain a positive-alpha bias. It derives an expression for the alpha of an asset allocator and examines the statistical properties of the estimated alpha for asset allocators with varying ranges of ability. It shows that the estimated alpha of an asset allocator is driven by two factors, one of which bears no relationship to ability and can be used to manipulate the measure such that even randomly chosen positions can lead to long-term positive estimated alphas. Only extremely large sample sizes, exceeding a manager's lifetime, can eliminate this bias. Alpha performs well in identifying managers with ability, but capable managers are intermingled with managers who can achieve positive estimated alphas using random number generators. Widely-used alternatives such as Henriksson-Merton and Treynor-Mazuy retain a significant risk of identifying ability where it does not exist, even in large samples.

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I. Introduction

One of the most widely-used concepts in investment management is the notion of an alpha or excess return generated by outperforming a benchmark index. Its theoretical basis is the classic Capital Asset Pricing Model (CAPM), wherein alpha is defined as a return that differs from the return the investor should earn as prescribed by the model. The CAPM is not necessary to define the existence of alpha, because any notion of an equilibrium return, that is, one merited by the risk assumed, establishes a benchmark against which alpha can be measured.[1] Of course, implementation is a more demanding task than it appears on the surface. The CAPM and its multifactor derivatives provide convenient, albeit sometimes controversial, benchmarks, although this family of models is unquestionably the most commonly used in determining alpha.

It is nonetheless virtually impossible to observe the investment industry in practice without noting the near obsession with the generation of alpha. Practitioner conferences are rife with presentations and panels on alpha, which is pursued like Indiana Jones chasing the Holy Grail. Practitioners debate whether asset allocation strategies can produce alpha and whether a performance is alpha driven or beta driven.[2] Of course, the explanation for this obsession is the fact that performance measurement, and consequently a significant element of compensation, is often tied to alpha. Unfortunately such compensation is usually based on short-term performance. Alpha, being a statistical measure, is clearly replete with random variation leading to positive noise, and hence positive alphas, that reward investment managers, while negative noise, and hence negative alphas, seldom reduce compensation but merely represent the forgoing of bonuses. Thus, for better or for worse, alpha is a firmly-implanted and widely-accepted concept in the investment community. Hence, it is important to have a solid understanding of the limitations of alpha so that it can at least be used where it is conceptually appropriate. One popular class of strategies where alpha is widely employed is asset allocation.

Asset allocation encompasses a family of strategies in which portfolios are constructed not by selecting individual assets in an efficient scheme but rather by optimally combining asset classes. Asset allocation is in some sense solidly grounded in CAPM theory, because the CAPM states that all investors would hold a combination of two asset classes, the risk-free asset and the market portfolio with the relative proportions of each class holding determined by investor risk preferences. While CAPM theory would suggest that the proportions allocated to the risk-free asset and market portfolio are relatively

[1]See Grinblatt and Titman (1989, p. 412), who argue that equilibrium models are need not be necessary for true performance measurement and only suggested candidates for particular benchmarks, such as mean-variance efficiency. Indeed, equilibrium models almost always assume market efficiency and hence disavow the existence of ability.
[2]See Blanchett (2010) for the former and Schneeweis, Crowder, and Kazemi (2010, Chapter 3) for the latter.

stable through time, asset allocation strategies typically involve changes from time to time, perhaps even quite frequently. The expression *dynamic asset allocation* has evolved to describe this practice.

Asset allocation strategies are frequently referred to as the practice of *strategic asset allocation*, which is the establishment of long-run allocations to the asset classes, or the practice of *tactical asset allocation*, which is the establishment of short-term deviations from the long-run allocations and are undertaken to take advantage of short-run information. Thus in tactical asset allocation, investment managers typically respond to short-run expectations of outperforming asset classes by deviating from long-run strategic allocations. Tactical asset allocation or TAA, thus, is essentially active management based on expectations of the relative performance of various asset classes. Although the CAPM establishes only two asset classes, multifactor models admit multiple asset classes that recognize tiers within markets (large, medium, small cap, value, growth), extend markets beyond borders (domestic, foreign/global, emerging), and define alternative asset classes (commodities, private equity, currencies, etc.).

Another strategy virtually identical to TAA but has suffered from the curse of its name is *market timing*, which is the process of actively moving in and out of the market based on short-term information.[3] If the asset classes are a broad equity market and a risk-free asset, then TAA cannot be distinguished from market timing. On a purely anecdotal basis, the terms *tactical asset allocation* and *market timing* imply virtually the same strategy but almost always elicit different reactions. Tactical asset allocation is viewed quite favorably while market timing is viewed quite unfavorably. For example, a Google search of the literal expressions "market timing does not (or doesn't) work" generated 291,200 hits, while the expression "tactical asset allocation does not (or doesn't) work" generated only nine hits. Broadening the expression to "asset allocation does not (or doesn't) work" produced 2,023 hits. While opponents of market timing are more vocal than opponents of asset allocation, proponents are apparently more vocal. The expression "market timing works (or does work)" produced 26,490 hits while "asset allocation works (or does work)" generated 32,980 hits. For this paper, we will use the term asset allocation but assume that it is equivalent to market timing.

The interest level in the strategy of asset allocation, whatever it may encompass, accelerated with the widely-cited Brinson, Hood, and Beebower (1986) paper that purported to show empirically that asset allocation accounts for about 94% of the variation in performance over time. Subsequent research by Brinson, Singer, and Beebower (1991), Vardharaj and Fabozzi (2007), Ibbotson and Kaplan (2000), Hensel, Ezra, and Ilkiw (1991), Kritzmann (2006), Ibbotson (2010), and Xiong, Ibbotson, Izdzorek and

[3]Market timing is sometimes distinguished from asset allocation by the fact that the former frequently refers to moving in and out of either the market or cash, while asset allocation is moving in and out of multiple risky asset classes and cash. Clearly, any type of asset allocation strategy is a market timing strategy and vice versa, but as we show here, what a strategy is called can make a huge difference in image,

Chen (2010) have re-examined and extended this issue to the questions of how these results vary by sample, within sample, and over time. It remains unclear how much of a particular portfolio's return is attributed to asset allocation, but interest in this topic remains high. In many ways, a resolution of this issue is fundamental to understanding capital markets. If in the aggregate, less than 100% of return performance is explained by asset allocation, then there are clearly systematic factors beyond the market portfolio that drive returns.

As we will later show formally, there is no inherent basis in the concept of alpha that supports the measurement of asset allocation ability. Alpha is a measure of selectivity. Common measures of asset allocation/timing are the noted Treynor-Mazuy (1966) and Henriksson-Merton (1981) regression betas. The Treynor-Mazuy measures derives from a regression of the excess portfolio return on the market index and the square of the market index, with the allocation/timing measure being the coefficient on the latter. The Henriksson-Merton measure derives from a regression of the excess portfolio return on the market index and a truncated market index return, with the latter being the greater of zero or the negative of the market return. Both measures theoretically capture the process by which good asset allocators shift their betas, moving up when the market is expected to be stronger and down when the market is expected to be weaker.

Tests of market timing and asset allocation ability have largely tended to show no ability. Most of these tests have been, quite naturally, applied to the returns of professional money managers such as mutual fund managers. Although we naturally wish to evaluate the actual data produced by professional money managers, such data can mask some of the true problems of measurement techniques. Most money managers, and virtually all mutual fund managers, engage in a multitude of strategies that mix asset allocation and security selection. Very few are pure asset allocators. Hence, true returns produced by managers are noisy and the techniques must find a way to separate asset allocation from selection.[4] Along the way, we encounter a number of conceptual difficulties. For example, timing measures can be negatively biased by the use of option-like securities (Jagannathan and Korajczyk (1986), Ingersoll, Spiegel, Goetzmann, and Welch (2007), and Hubner (2010)). Returns are generated by an unknown process that requires the specification of one or more factors. Any such test of performance is, therefore, a joint test of performance and the number and identify of the return-generating factors. All managers have access to a common set of information but some managers may have additional information or special ability. Accurate performance measurement would identify those managers whose performance is

[4]Two exceptions are Graham and Harvey (1966) and Chance and Hemler (2001). The former examine investment newsletters and find poor performance among market prognosticators. The latter examine actual trade-by-trade data of a group of managers who present themselves strictly as market timers/asset allocators. They find strong performance, which appears only in daily but not monthly data.

beyond that which could be produced by using well-known market information. Hence, tests must be conditioned on the information available to all managers.

In this paper we focus strictly on asset allocation and use simulated data produced by hypothetical asset allocation strategies that are by constructed by either a completely random decision rule or by a pre-specified forecasting ability. These strategies are applied to a market with only an equity index and a risk-free asset, both of which have stationary parameters. Hence, there is no contamination of asset allocation with selectivity nor are there any derivatives that can be used to manipulate performance measurement. There is no need to apply any conditional measures of performance, since returns are white noise and cannot be predicted from macro-economic or –financial information. A multi-factor approach is clearly not required. Hence, all of the real-world imperfections that must be accounted for in empirical data are sanitized from this experiment. If the asset allocation performance measure fails to detect ability in hypothetical managers pre-specified to have foresight and/or attributes ability to managers who use random decision rules, then we know the performance measure completely fails and should not be used in a less-than-perfect real environment.

As noted, our objective is to examine alpha as a performance measure but other measures could be tested in a similar manner. Alpha is conceptually related only to security selection, but in practice it is widely used to evaluate asset allocation ability. There is substantial evidence that practitioners believe that asset allocation can be a source of alpha. Many well-known Wall Street firms and other money management firms and individual professional managers regularly claim that their asset allocation strategies either generate or are designed to generate alpha. Many firms report statistics on alphas they generate from these strategies. There are several hundred funds classified as "asset allocation funds" and many of them are evaluated by analysts by examining their alphas.[5]

The objective of this paper is to examine the relationship of alpha to asset allocation and to show that positive alphas that are so frequently claimed to be generated by asset allocators are a statistical artifact. They are not, however, generated by noise, which would be as likely to produce negative alphas as positive alphas. Rather, we show that the application of alpha theory and empirics to asset allocation strategies will lead to a tendency for alphas to be positive. Unfortunately, we also show that it is virtually impossible to correct this bias even in large samples. Hence, asset allocators with no ability will often be lumped in with those who do have ability. From that result, it is simple to see that the investment profession will be motivated to continue the use of alpha in evaluating asset allocation ability. Hence, an awareness of this problem can at least lead to a modest degree of skepticism when considering asset allocation performance.

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[5]Because the principal objective of this paper is to show that alpha is an extremely misleading if not dangerous device for measuring asset allocation ability, we will avoid specific references to these firms, but examples can be provided upon request.

The paper is organized as follows. Section II illustrates in a simple, conceptual framework why alpha is not a measure of asset allocation ability. Section III introduces randomness into a simple example of a contest between multiple managers. Section IV examines the large-sample statistical properties of alphas for asset allocators. Section V presents a simulation of hypothetical managers. Section VI presents the conclusions.

II. The Fallacy of Alpha in a Conceptual Framework

First we establish a definition of alpha. We will operate in the CAPM world with two asset classes, a broad market index, referred to as the market, and a risk-free asset.[6] Of course the well-known CAPM is

$$E(R) = r + [E(R_m) - r]\beta,$$

where $E(R)$ is the expected return on the portfolio, r is the risk-free rate, $E(R_m)$ is the expected return on the market portfolio, and β is the beta or covariance risk of the portfolio with the market. Alpha (α) is typically defined as a deviation of the actual portfolio return from the expected return. Given that the actual market return could be negative, however, the expected return is typically adjusted after the fact so that the realized market return and realized portfolio return are used to proxy the expected return. With beta estimated in the traditional times series manner as $\hat{\beta}$, we let $\bar{R}$ be the realized portfolio return and $\bar{R}_m$ be the realized market return. The estimated alpha is defined as[7]

$$\hat{\alpha} = \bar{R} - [r + (\bar{R}_m - r)\hat{\beta}].$$

Now let us proceed to see how the estimated alpha performs as it attempts to distinguish a well-performing asset allocator from a poorly-performing one.

Consider two investment managers who practice asset allocation. One is an optimist and invests 100% in a stock index fund that is assumed to be a good proxy for the market. The other is a pessimist and invests 100% in the risk-free asset, which we will refer to as cash. Now, suppose the index fund earns 10% and cash earns 3%. The optimist clearly has a beta of 1.0 and earns 10%, while the pessimist clearly has a beta of 0.0 and earns 3%. Their alphas are as follows:

Optimist: 0.10 - [0.03 + (0.10 - 0.03)1.0] = 0.0

Pessimist: 0.03 - [0.03 + (0.10 - 0.03)0.0] = 0.0

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[6]The extension of all of the issues addressed in this paper to world of more than two asset classes should in no way reduce the problems with alpha illustrated here. Indeed, it is likely to compound the problems, because the manager now has to choose the best performing asset class from at least three classes.

[7]Of course, we are also clearly assuming stationarity of the risk-free rate and the beta. These are separate and important issues, but we do not directly address them here. Later, however, the beta-stability question is tackled. Asset allocators, by definition, are frequently shifting their betas, which has important implications for alpha measurement.

As we see, alpha is unable to distinguish these two managers. The optimist correctly predicted that the market would outperform cash and invested accordingly but ends up with the same alpha as the pessimist. It is easy to see that any outcome will produce the same alpha for all asset allocators.[8]

The intuition should be obvious. A manager who invests only in stock or cash or a combination of both is simply sliding up and down the security market line. Alpha is, by definition, a deviation from the security market line. A manager cannot generate an alpha without moving off the security market line. Thus, it is not possible to produce alpha using an asset allocation strategy.

This example illustrates the conceptual framework but does not capture certain important elements introduced when one must collect data and statistically estimate alpha. We will establish these properties in a gradual manner. First we build a simple two-period model, illustrate certain key statistical calculations, and then derive the results in a framework of multiple periods.

III. Statistical Problems in the Estimation of Alphas for Asset Allocators

In a one-period model as in the above section, there are no statistical complications. More critically, however, the beta chosen by the manager is the beta of the portfolio over the full time period examined. When a second period is added, however, the beta could change. This raises the question of what the portfolio beta is over the entire time period. Of course, that beta must be estimated and will contain statistical error.

We will assume a two-period world with two asset classes. In the first period, a market index can move up 20% or down 10%. In the second period, it can move up 25% or down 20%. The probability of an up move is 0.65. The cash account pays 3% each period.[9] Under these assumptions the market expected return is 15.88% and its volatility is 30.13%. Figure 1 illustrates the value of an initial $1 investment for all outcomes.

Consider four investment managers. Manager 1 is a buy-and-hold manager who invests in the index fund today and holds the position through both periods. This manager's performance will clearly mirror that of the index. Manager 2 is an asset allocator who believes that in the first period the index will beat cash but that in the second period, cash will beat the index. So Manager 2 invests in the market the first period and in cash the second. Manager 3 is an asset allocators with the opposite view and invests in cash the first period and in the index the second period. Manager 4 believes that cash will beat the index both periods, so he invests in cash both periods. His performance will mirror that of a money market fund.

Exhibit 2 shows the cumulative value of $1.00 initially invested by all four managers. This framework will be sufficient to enable us to see how statistical analysis on historical data can reveal a

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[8]The math is very simple. The return on portfolio i with beta of β_i is $\beta_i R_m + (1 - \beta_i)r$. Substituting into $\alpha_i = R_i - [r + (R_m - r)\beta_i$ gives a value of zero for any return.

[9]Of course, this framework is that used in the binomial option pricing model, but no derivatives are required here.

non-zero estimated alpha. Unfortunately, it will not help us determine who did the better job of asset allocation.

Suppose we perform a statistical analysis of the performance of these four managers. Obviously a time series statistical sample is a sequence of outcomes. Since there are four distinct sets of outcomes for each example, we must consider what conclusion we ought to draw contingent upon the four outcomes occurring.

The four outcomes are

Outcome 1: Market up 20%, market up 25%

Outcome 2: Market up 20%, market down 20%

Outcome 3: Market down 10%, market up 25%

Outcome 4: Market down 10%, market down 20%

If outcome 1, occurs we should conclude that Manager 1 had the best performance. The market went up sufficiently in both periods to outperform cash and Manager 1 was the only manager in the market both periods. If outcome 2 occurs, we should conclude that Manager 2 had the best performance. The market first went up sufficiently to outperform cash and then went down. Manager 2 was in the market the first period and in cash the second period. If outcome 3 occurs, we should conclude that Manager 3 had the best performance. The market first went down and then up sufficiently to outperform cash. Manager 3 was in cash the first period and then in the market the second period. If outcome 4 occurs, we should conclude that Manager 4 had the best performance. The market went down both periods, and Manager 4 was in cash both periods. Thus, it is obvious who the winners are. Now let us see what a statistical analysis would conclude.

There is a possibility of one of four sample outcomes. For each sample we will have two observations, from which we can estimate a beta. Knowing the return on the market and the risk-free rate, we can obviously estimate an alpha. Let us work through one example.

Consider Manager 2, outcome 3. Manager 2 loses 10% the first period and earns 3% the second. This is a sample average of (-10% + 3%)/2 = -3.5%. The variance of Manager 2's return is

$$[(-0.10 - - 0.035)^2 + (0.03 - - 0.035)^2]/1 = 0.0085.$$

We must also determine the mean and variance for the market. Based on returns of -10% and 25%, these results are 0.075 (mean) and 0.0613 (variance).

The covariance of Manager 2 with the market is

$$[(-0.10 - -0.035)(-0.10 - 0.075) + (0.03 - -0.035)(0.25 - 0.075)]/1 = 0.0228.$$

Therefore, the estimated beta of Manager 2 is

$$0.0228/0.0613 = 0.3714.$$

The average return on the market is 7.5% and the risk-free rate is 3%. Thus, the estimated alpha for Manager 2 if outcome 3 occurs would be

$$-0.035 - [0.03 + (0.075 - 0.03)(0.3714)] = -0.0817.$$

Following the same procedure, the estimated alphas of the four managers for outcome 3 are

Manager 1: 0.0000

Manager 2: -0.0817

Manager 3: 0.0817

Manager 4: 0.0000

These results look promising for alpha as a performance measure. We rationalized that Manager 3 should be the winner. The estimated alpha agrees with that conclusion.

Table 1 provides the estimated alphas for all four managers and all four outcomes. Now we see, however, that the best estimated alphas do not reflect the obvious winners. We noted that Manager 1 should be the winner if outcome 1 occurs. Manager 1 is in the market each period and the outcome is the case of the market return exceeding cash each period. Yet, Manager 2 has the highest estimated alpha, and Manager 1 has an estimated alpha of 0.0. Manager 2 was in the market in the first period but was in cash the second. Also notice outcome 4 in which the market went down both periods. Manager 4, in cash both periods, should be the winner, but Manager 3, who was wrong the second period, has the highest estimated alpha.

This example begins to illustrate the problem of using alpha to measure asset allocation performance. Alpha seems to have little to do with actual asset allocation decisions. It is much more influenced by statistical issues. For example, for each of the four outcomes, Manager 1, who is in the market both periods, has a beta of 1.0. Likewise, Manager 4, who is out of the market both periods, has a beta of 0.0. Manager 2 who is in the market the first period and out the second has betas of -3.4, 0.425, 0.3714, and -1.3000 for outcomes 1 through 4, respectively. Manager 3 who is out of the market the first period and in the market the second has betas of 4.4, 0.5750, 0.6286, and 2.3 each of the four outcomes. Why the difference? The first period market return was 0.20 or -0.10, and the second period market return was 0.25 or -0.20. Obviously market volatility has a significant impact on the statistical estimation of betas.

The behavior of the estimated alpha for large samples can be ascertained, which we do in the following section.

IV. Estimated Alphas for Asset Allocators with Large Samples of Returns

Actual performance evaluations involve the estimation of alpha using more than two returns. In this section we see what would happen to the estimated alpha in a large sample of returns.

A. Basic Definitions

Again we assume two asset classes, one a market index and the other a cash or risk-free account. We observe a sequence of periodic returns on a market index,

$$R_{m1}, R_{m2}, \ldots R_{mT},$$

which can be expressed, as R_{mt}, $t = 1, 2, \ldots, T$. For simplicity, let us treat each period as a day. There is a constant daily risk-free rate r. Each day an asset allocator will choose whether to be in or out of the market, thereby setting his beta at either 1 or 0 each day. The days in which the manager is in the market will be referred to as the *exposure period,* and the days in which the manager is out of the market will be referred to as the *hedge period*. Having observed the market performance and the investment manager's positions, we note that there are T_i days in which the manager is in the market, with a beta of 1.0, and T_o days in which the manager is out of the market with a beta of 0.0. Thus, $T_i + T_o = T$. The overall set of market returns can be divided into those that occur during the manager's exposure period and those that occur during the manager's hedge period,

$$\begin{array}{ll} R_{mt}(i),\ t = 1, 2, \ldots T_i & (\textit{the exposure period}) \\ R_{mt}(o),\ t = 1, 2, \ldots T_o & (\textit{the hedge period}). \end{array}$$

The series of returns for the manager can also be subdivided into those that occur during the exposure period and those that occur during the hedge period. These returns will not correspond to those of the market for the hedge period, because in that case the manager will earn the risk-free rate. The returns during the exposure period are the market returns,

$$R_t(i) = R_{mt}(i),$$

for all $t = 1, 2, \ldots, T_i$, and the returns during the hedge period are the risk-free rate,

$$R_t(o) = r,$$

for all $t = 1, 2, \ldots, T_o$.

From this data, certain statistics will be estimated. For example, we would want to estimate the average return on the market:

$$\overline{R}_m = \frac{\sum_{t=1}^{T} R_{mt}}{T}.$$

We would also want to know the average return on the market during the exposure period and the hedge period. Define p = Let T_i/T as the proportion of days the manager is in the market and refer to this as our measure of the exposure period. We will call this measure the *exposure*. Then $1 - p = T_0/T$. It follows that the average market return is a weighted average of the average market return for the exposure period and for the hedge period,

$$\overline{R}_m = \overline{R}_m(i)p + \overline{R}_m(o)(1-p). \tag{1}$$

B. Specification of the Estimated Alpha

Alpha is estimated by subtracting the sum of the estimated market risk premium and risk-free rate from the manager's return,

$$\hat{\alpha} = \overline{R} - (r + (\overline{R}_m - r)\hat{\beta}).$$

Using our breakdown of the manager's return into its components during the exposure and hedge periods, as well as the market return during the exposure and hedge periods, we can specify the estimated alpha as follows,

$$\begin{aligned}\hat{\alpha} &= p\overline{R}_m(i) + (1-p)r - (r + (\overline{R}_m - r)\hat{\beta}) \\ &= p\overline{R}_m(i) - \hat{\beta}\overline{R}_m + r(\hat{\beta} - p) \\ &= (p - \hat{\beta})(\overline{R}_m - r) + p(\overline{R}_m(i) - \overline{R}_m)\end{aligned} \tag{2}$$

Now we see that the estimated alpha of a manager has two components. The first, $(p-\hat{\beta})(\overline{R}_m - r)$, is the market risk premium multiplied by the difference between the exposure and the estimated beta. This term is problematic as it has nothing to do with skill. Though being in the market at the right time is critical, the exposure is not a skill factor. The manager could have a relatively high exposure by being in the market at precisely the wrong time or just the opposite. In addition, the market risk premium is clearly unrelated to the manager's skill. This first term can, however, be zero, which obviously occurs if the exposure equals the estimated beta. In other words, if a manager is in the market 60% of the time and his estimated beta is 0.6, then this term goes to zero. As we will show later, the exposure is closely related but not necessarily equal to the estimated beta except under certain conditions that we will identify.[10]

The second term, $p(\overline{R}_m(i) - \overline{R}_m)$, is, however, clearly related to skill. The proportion of time the manager is in the market is multiplied by the difference between the market average return during the days in which the manager is in the market and the market average return over all days. If the manager is

[10]Our equation (2) is similar to the Equation (4) of Grinblatt-Titman (GT) (1989) but differs in focus and extension. As shown in their Fig. 1, the alpha of a manager selecting positive but varying betas will be biased negatively. GT's objective is to find a means of removing the negative alpha bias when timing is intermingled with selectivity, and they do so by focusing on the weights of the component securities. We consider the case of a manager choosing only a beta of zero of 1. An analogous version of GT's Fig. 1 would also confirm a negative alpha. GT, however, focus only on the case that the manager reacts to a signal of low or high market excess returns that are both positive. If the low return were negative, which would be the case if the risk-free rate is expected to outperform the market, then GT's Fig. 1 would clearly show a positive alpha. Also, as GT show, the timing bias goes to zero in large samples, a result we confirm here, but we show that such sample sizes are not possible in reality.

in the market when it is performing well, then $\bar{R}_m(i) - \bar{R}_m$ should be positive. If that is true, then the more often the manager is in the market (higher *p*), the larger is the manager's estimated alpha. The manager does not, however, directly determine p, which is simply a consequence of the quality of the decision rule he makes along with the relative frequency with which the market outperforms the risk-free rate.

An alternative expression for the estimated alpha can be derived by substituting Equation (1) into Equation (2) to obtain

$$\hat{\alpha} = (p - \hat{\beta})(\bar{R}_m - r) + p(1-p)(\bar{R}_m(i) - \bar{R}_m(o)). \tag{3}$$

This equation differs from the previous one in only the second term, which highlights the difference in the market's performance during the exposure and hedge periods. In both equations, it is apparent that the second term measures ability, while the first contributes to the estimated alpha but does not measure ability. Moreover, it is apparent that if $p > \hat{\beta}$ and the market risk premium is positive, the estimated alpha will be biased upward.

Thus, it is important that we understand the relationship between *p* and $\hat{\beta}$. In the Appendix we show that the estimated beta of the two-fund asset allocator is

$$\hat{\beta} = \frac{Tp-1}{T-2} - \frac{(\bar{R}_m - r)T_o p(\bar{R}_m(o) - \bar{R}_m(i))}{\sum_{t=1}^{T_i}(R_{mt}(i) - \bar{R}_m)^2 + \sum_{t=1}^{T_o}(R_{mt}(o) - \bar{R}_m)^2} \tag{4}$$

While we will examine the components of this expression thoroughly in the next sub-section, note that it contains one simple term related to *p* and the sample size *T* and one more complex expression that somewhat resembles a type of covariance divided by the variance of the market.

Let us now examine the case of a neutral manager, that is, one with no ability. We will examine the conditions under which the estimated alpha correctly captures this manager's performance. If the estimated alpha is an accurate measure, such a manager should have an estimated alpha of zero.

C. Managers without Ability

We propose a simple characterization of a manager with no ability: *a manager with no asset allocation ability will produce an average return during the exposure period equal to the average return during the hedge period, which by definition is the same as the average overall market return*. That is, for the manager with no ability,

$$\bar{R}_m(i) = \bar{R}_m(o) = \bar{R}_m \tag{5}$$

This definition should make intuitive sense. For a manager who is unable to forecast which asset class will perform better, the market will perform no differently while he is in the market than it will while he is out of the market. Thus, for this manager the estimated alpha reduces to

$$\hat{\alpha} = (p - \hat{\beta})(\overline{R}_m - r) .$$

If $p = \hat{\beta}$, we have no problem as the estimated alpha will go to zero, as it should.

Now we need to determine the manager's estimated beta and see how it relates to *p*, the exposure. Using Equation (5) in Equation (4) gives us a simple expression for $\hat{\beta}$,

$$\hat{\beta} = \frac{Tp - 1}{T - 2} \tag{6}$$

for the manager with no ability. We would of course like to find that $\hat{\beta} = p$, leaving $\hat{\alpha} = 0$. In one simple case, that will be true, which is that *p* = ½, but we need to look further at what *p* can or will be for the manager with no ability.

A reasonable characterization of a manager with no ability would be that the manager chooses his positions randomly. Thus, a random number generator could be used to select whether to be in the market or cash. Nonetheless, the manager would need some type of criterion. A reasonable decision rule might be to generate a unit uniform random number and be in the market when the random number exceeds 0.5 and in cash otherwise. In that case, *p* = ½, which reduces the estimated alpha to zero. If the manager were aware of this point, *p* = ½ would be a poor choice of exposure.

But there are many other reasonable exposures one might select if one were choosing positions at random. The manager could choose to be in the market 75% of the time overall and thus he would go into the market when the random number is less than 0.75. It is impossible to establish a random decision rule without specifying a cutoff. Thus, the manager with no ability is forced to choose *p*.

Figure 3 shows the relationship between *p* and $\hat{\beta}$ for various sample sizes. We see that p > (<) $\hat{\beta}$ for exposures of less (greater) than 0.5. Hence, with a positive risk premium, a manager with no ability would do well to choose relatively low exposures. Clearly his estimated alpha will be a positively biased measure of his ability, as he will generate positive estimated alphas if the market risk premium is positive.

This problem is mitigated somewhat by sample size as the difference between *p* and $\hat{\beta}$ diminishes with larger sample. But as we see here, the relatively small samples that are typically used in practice (say, 250 days or one year), are not sufficient to eliminate the bias.

As an example, consider a manager who trades only yearly over the 1960-2007 period. We are not allowing for variation in the risk-free rate so let us just use the average rate over that time of 5.9%. There are 47 periods in that era. Let the manager be in the market just 10% of the time with his exposure determined using a random number generator. The manager's estimated beta should be

$$\hat{\beta} = \frac{47(0.1)-1}{47-2} = 0.08$$

Thus, he is in the market 10% of the time and his estimated beta is 0.08. The average market return was 8.3%. Thus, his estimated alpha would be

$$\hat{\alpha} = (0.083 - 0.059)(0.10 - 0.08) = 0.00048.$$

He earns roughly five basis points being in the market 10% of the time, with his positions chosen using a random number generator.

Thus, a completely naïve strategy of using a random number generator that will put the manager out of the market more than he is in can produce a positive long-run estimated alpha of about five basis points, provided that stocks beat the risk-free rate over the measurement period. Moreover, if the probability is high that stocks will beat cash, such a strategy will lead to poorer forecasting accuracy. Nonetheless, such a manager, if evaluated by the estimated alpha, will be rewarded.

It is important to see how a positive estimated alpha has no bearing on forecasting accuracy. Let p_G = the proportion of markets that are said to be "good," that is, where stocks beat cash. The manager's forecasting accuracy or percentage of correct forecasts, p_c, is by definition

$$p_c = pp_G + (1-p)(1-p_G).$$

To analyze that expression, we need to know a reasonable range of p_G, the probability that stocks will beat cash. Under the assumptions of a lognormally distributed index, an expected return ranging from 6% to 20%, volatility ranging from 5% to 50%, and a risk-free rate ranging from 2% to 8%, the probabilities of stock beating cash vary from 48% to 58%, though concentrated around 50%. Thus, we consider a range of p_G of 40% to 60% and p of from 25% to 75%.

The results are presented in Table 2. First note that if the probability that stocks beat cash is 50%, any value of p will produce accuracy of 50%. If the probability that stocks beat cash is more than 50%, the manager's accuracy is lower, the lower is p. Hence, managers that attempt to game their estimated alphas produce a lower level of forecasting accuracy and arguably are therefore, poorer performers, but according to the estimated alpha, they may show as excellent performers. Only if the probability of stocks beating cash is low will their accuracy be somewhat reflected in their estimated alphas. Lower exposure will lead to higher accuracy and higher estimated alphas.

Figures 4a and 4b illustrate the estimated alpha for a manager for varying levels of exposure and how that estimated alpha varies with the level of the market return and the sample size. We assume that regardless of the level of the market return, the average market return while the manager is in the market is the same as that while the manager is out of the market, the condition that defines the absence of ability. In Figure 4a, we see, as previously mentioned, that when the market is performing poorly, positive (negative) estimated alphas are obtained for high (low) levels of exposure. When the market is performing well, positive (negative) estimated alphas are obtained for low (high) levels of exposure. Accordingly, an asset allocator, figuring on a positive market the majority of times in his career, can maintain a low level of exposure and virtually guarantee positive estimated alphas for the majority of his career. Figure 4b shows that sample size somewhat mitigates this bias. The estimated alpha monotonically approaches zero as the sample size increases. Unfortunately, in practice, large samples are rarely used. In fact, a sample of 10,000 trading days would take virtually an entire 40-year career. As the figure shows, substantial positive estimated alphas can be obtained over a period as short as 1,000 days or about four years.

D. Managers with Ability

Now we consider the more complex case of the manager with ability. We want to know if the estimated alpha can identify this type of manager. We already know that the estimated alpha can label managers with no ability as having ability. But will it recognize managers with ability and assign them positive estimated alphas?

First, we have to define ability, and that is not an easy task. We cannot simply define it as the condition that the average market return when the manager is in the market exceeds the risk-free rate. Someone in the market all the time would have that likely to be true. Given that we identified no ability as the condition that the return on the market while the manager is in the market equals the return on the market while the manager is out of the market, we may have a reasonable basis for identifying ability in a similar manner. Therefore, as a first step, we define a necessary condition for asset allocation ability to be

$$\overline{R}_m(i) > \overline{R}_m(o), \tag{7}$$

with the corresponding measure,

$$\overline{R}_m(i) - \overline{R}_m(o). \tag{8}$$

Heuristically, condition (7) seems necessary. There is little question that a manager without ability, who by definition cannot predict how the market will behave, should perform no differently when out of the market than when in the market. Hence, a manager with ability should perform better when in the market than when out. Nonetheless, meeting this condition is unlikely to be sufficient nor is it likely to be a very discriminating condition. Consider a manager who is in the market for only a small number of periods with the largest positive returns. His average return while in the market will clearly exceed his average return while out of the market, but we are unlikely to think he is much of a manager. He failed to be in the market during many periods in which the market outperformed the risk-free asset and hence, he missed many good opportunities.

Consider a set of observed market returns of 15%, 7%, 5%, -5%, and -7%, and a risk-free rate of 4%, and a set of six managers.[11] Manager 1 correctly calls the first, second, and fifth markets. Manager 2 correctly calls the first, second, and fourth markets. Manager 3 correctly calls each market except the first. Manager 4 correctly calls the first, fourth, and fifth markets. Manager 5 correctly calls each market except the fifth. Manager 6 correctly calls each market except the third. For Managers 1, 2, and 4 who correctly call 60% of the markets, the return differential in Equation (8) is highest for Manager 4 and lowest for Manager 2.[12] For Managers 3, 5, and 6 the return differential is highest for Manager 6 and lowest for Manager 3. Managers 2 and 3 have the same return differential, but Manager 3 correctly called 80% of the markets, while Manager 2 correctly called 60% of the markets. The highest return differential is Manager 4, but he correctly called only 60% of the markets, while three other managers correctly called 80% of the markets. Thus, the return differential, which of necessity has to be positive for managers with ability, is not sufficient to discriminate.

The return differential is not, however, without merit. The manager who calls perhaps the best performing positive markets would appear to have important ability. If the performance in the positive markets is strong enough, it could offset the failure to correctly call many other markets. The problem is that we have no clear-cut measure of performance for managers whose betas are shifting. A CAPM-based measure such as an estimated alpha, as we showed earlier, can take managers with no ability (i.e., those using a random number generator) and make it appear that they have ability.

Given the unsettled state of knowledge on this problem, we will proceed with ability defined by accuracy, the percentage of markets correctly called with the necessary condition of Equation (7). An important question is then how accuracy converts into exposure. By definition, the exposure of a manager is the proportion of good market periods times the probability the manager makes the correct call plus the proportion of bad market periods times the probability the manager makes an incorrect call. Then by definition, p is

$$p = p_G p_c + (1 - p_G)(1 - p_c).$$

Table 3 presents values of p for a range of values of p_G and p_c, with p_G varying as previously described. We see that the exposure ranges from 40% to 60% and is concentrated around 50%. For example, if the

[11]The returns are in order from highest to lowest but the actual sequence is not important.
[12]Detailed calculations are available on request.

probability of a good market is 52%, note that a manager who is correct 50% of the time will be in the market 50% of the time, while a manager who appears to be much better by being correct 75% of the time will be in the market only 51% of the time. Even if the probability of a good market is 60%, the manager with 50% accuracy is in the market 50% of the time, while the manager with 75% accuracy is in the market only 55% of the time. Thus, we see that over a wide range of manager accuracy, the exposure varies very little, between 45 and 55% and is concentrated around 50%.

We next take a look at how the estimated alpha relates to the various inputs. We assume a risk-free rate of 3% and 250 trading periods. In Figure 5a, we let the average market return be 9% and market volatility be 20%. We vary *p* from 5% to 95% but focus on values close to 50%. Figure 5a examines the estimated alpha as it relates to exposure and the return differential ranging from 2% to 10%. We see that the estimated alpha is directly related to the return differential, regardless of the exposure. Moreover, the larger the return differential, the higher is the estimated alpha.

With the return differential fixed at 4%, Figure 5b shows the estimated alpha as a function of the level or average return of the market. The estimated alpha is generally higher with the higher level of the market. While the difference is very small at low levels of exposure, we noted previously that most exposures are likely to be around 0.5.

Figure 5c examines the effect of sample size, which we vary from 100 to 100,000. We see that larger samples have little effect on the estimated alpha. This may perhaps be comforting, as it suggests that the estimated alpha could be an accurate measure of ability for managers with ability, even though alpha is not technically a measure of asset allocation ability. Even large sample sizes do not hinder the ability of the estimated alpha to detect manager quality. In Figure 5d, we see the effect of different levels of market volatility. The effect is small at low levels of exposure, but as we noted, low levels of exposure are unlikely for managers with ability.

V. Simulations of Asset Allocation Performance

To further explore these relationships in a dynamic market environment, we conduct several Monte Carlo simulations. We first construct a set of hypothetical managers following diverse strategies and with diverse skills. Table 4 describes these managers. Manager 1 is always in the market and Manager 5 is always in the risk-free asset. Managers 2-4 have different pre-specified exposures, which are 75% for Manager 2, 50% for Manager 3, and 25% for Manager 4. For a given day, whether these managers are in the market or out is determined by a random number generator with the threshold set such that the respective probabilities of being in the market are 0.75, 0.50, and 0.25, respectively. Obviously these managers have no ability and provide a benchmark for whether the estimated alphas of these managers will be zero.

Next we construct a set of managers with varying degrees of ability. Manager 6 is assumed to be in the better performing asset class 100% of the time. Obviously this manager has perfect foresight and would not exist in reality but provides an upper benchmark. Managers 7-12 are in the better performing asset class 70%, 65%, 60%, 55%, 50%, and 45% of the time, respectively. These percentages are, thus, measures of the accuracy of these managers. For simulation purposes, their positions are determined in a two-step manner. First, for each day the simulation identifies the better performing asset class. Then for each of these managers it generates a random number between 0 and 1. If the number is below (above) the accuracy for the manager, we assume the manager is in the better (worse) performing asset class. These managers will serve as a means of examining the properties of the estimated betas and alphas for managers with different degrees of skill.[13]

We set the average return on the market to 12%, the volatility to 20%, and the risk-free rate to 5%. Given these figures, the probability that stocks beat cash is between 50 and 51%. Each trading period is a day, and each day the manager can change the position from market to cash or vice versa as discussed above. Each simulation run is 250 days or approximately one year. The market index is generated with a geometric Brownian motion process, as is commonly used in pricing derivatives.

A. Large Sample Results

Table 5 presents summary statistics that represent averages of the results for one million 250-day periods. As a reality check, note that the average return for Manager 1, who is in the market 100% of the time, is 12% as specified in the simulation inputs. In addition, that manager's volatility is approximately 20%, the input volatility. In addition, Manager 5, who is out of the market 100% of the time has an average return equal to the specified risk-free rate of 5% and a volatility of 0.0%. Recall that Managers 2-4 choose their positions using random generators with thresholds set to put them in the market 75%, 50%, and 25% of the time, respectively. As the exposure column indicates, this result is obtained. Note in particular Column 7, which is the difference between the market return during the exposure period and the market return during hedge period. We see that when managers have no asset allocation ability, the market would perform no differently while they are in the market than it would while they are out of the market. Though not shown here, the volatilities of all managers during their exposure periods are approximately the same as the market volatility, indicating that as we assumed, there is no implicit volatility timing. We also observe that the accuracies of these managers who have no ability are all about 50%, as we demonstrated in Table 2 would be the case.

Recall that the estimated alpha also consists of a first term (note Equation (2)), which has nothing to do with ability. With a positive market risk premium, we found that low exposure can lead to a

[13]We do not define a particular level as being indicating ability or the absence of it, but some insights into this question are provided by this set of managers. In particular, we can focus on relative ability. Manager 7 should be better than Manager 8, who should be better than Manager 9, etc.

positive estimated alpha. Hence, someone like Manager 4 might have a chance to produce a positive estimated alpha. In this case, however, the sample size is sufficient to eliminate this possibility.

Thus, as we see over this large sample of one million 250-day periods, the estimated alphas of Managers 2-5, who have no ability, are all approximately 0.0%. For the managers imbued with varying degrees of ability, their estimated alphas line up according to their respective abilities. Recall that Managers 6-12 are structured to be accurate 100%, 70%, 65%, 60%, 55%, 50%, and 45%. Since, as we showed in Table 3 and verify here, their exposures will not vary much from 50%. Thus, their estimated alphas will be heavily driven by the return differential, the difference in the columns labeled $\bar{R}(i)$ and $\bar{R}(o)$. Hence, we see that the return differential is consistent with accuracy, but this is not surprising. We did not create any managers whose ability is concentrated in certain levels of the market.

Again, the effects of the first term in the equation for the estimated alpha are neutralized. As we showed in Figure 3, exposure of around 50% is sufficient to eliminate the effect of this term for managers with ability, though for managers without ability this term can lead to a bias. Thus, regardless of the sample size, managers with ability will be in the market about 50% of the time and this will eliminate the bias of the first term in the expression for the estimated alpha.

But there is yet another problem. The simulation is a large sample test. We executed one million 250-day runs. Clearly we must consider the variation from run to run. As such, the simulation also generates standard deviations of all of the sample averages. To conserve space, these are not shown but we discuss them here. We are particularly interested in the standard deviations of the estimated alphas. For the managers with no ability, these range from 8.6% to 9.8%. Thus, about two-thirds of the time their estimated alphas can vary plus or minus 8%, an enormous range. For the managers with ability, their estimated alphas also have a range roughly averaging more than 9%. These figures raise further doubts about the stability and consistency of whatever accuracy estimated alpha may appear to have. We next look at a simulation that more closely corresponds to realistic conditions.

B. Small Sample Results

We next conduct a simulation over a 40-year period, which would roughly correspond to a manager's career. Thus, we run 40 trials of 250-day periods. The results are presented in Table 6. A 40-year period does appear to be sufficient for certain desired limiting conditions to be met. The exposures of the managers with no ability are approximately equal to their pre-specified exposures. Note, however, that our principal measure of ability, the difference in the average market return while the manager is in and out of the market, is not zero for these managers. As such their estimated alphas are not necessarily zero. Note in particular, Managers 2 and 3 whose strategies are simply to be in the market 50% and 25% of the time. They generate average estimated alphas of about 2.6% and 1.6%, respectively. Although the theory suggests they should show no positive estimated alphas, the sample size is not sufficiently large to properly discriminate. For example, the differences between the market returns when these managers are in and out of the market are not zero. For Manager 3, with an estimated alpha of about 2.6%, the market earned 10.73% more while that manager was in the market than when he was out of the market. For Manager 4, with an estimated alpha of about 1.6%, the market earned 8.02% more while he was in the market than when he was out of the market. If these were managers evaluated over a 40-year period, such as their entire careers, they would be considered excellent managers. How surprising it would be if Manager 2, who is in the market half of the time, revealed at his retirement dinners that he had been using a coin toss to decide whether to be in the market. The 40-year period does seem sufficient to replicate to a sufficient degree the respective pre-specified accuracies of Managers 6-12, at least on average. In this case, the differences in the average market returns while these managers are in and out of the market align according to their abilities. As such, their estimated alphas also align correctly. Thus, if a manager has ability and is evaluated over his entire lifetime, that ability will likely be detected by the estimated alpha, at least on average. Unfortunately, some managers with no ability may also be evaluated as having ability. Moreover, the standard deviations remain large, more than 7% for the managers with no ability and more than 8% for the managers with ability.

This 40-year simulation produced an average market return of about 15%, which is somewhat high relative to the pre-specified expected market return of 12%. Nonetheless, such outcomes can occur. As a check, a second 40-year simulation was conducted and it produced an average market return of 11.6%. Manager 2 generated an average estimated alpha of 1.1%, while Manager 3 had an average estimated alpha of 0.5% and Manager 4's estimated alpha was -1.88%. Thus, even a period of a manager's lifetime is not sufficient to generate estimated alphas of zero for managers who choose their positions randomly.

Thus, we see there are problems in evaluating an asset allocation money manager over a lifetime. Of course, in practice evaluation is done over a much shorter period. Let us put this question to the test by conducting a single run of 250-days, thus approximating one year. Table 7 presents these results.[14] First note that the pre-specified exposures of Managers 2-4 and the accuracies of Managers 6-12 are approximately achieved in this very short run test. The market return, however, was 15.21%, which is considerably higher than the pre-specified return of 12%. Note that the estimated betas do deviate somewhat from the exposures and the average market returns during the managers' exposure periods deviate substantially from those in their hedge periods. Note now that Manager 3 has an estimated alpha of -8%, while Manager 4 has an estimated alpha of more than 8%. For the managers with varying degrees of ability, the estimated alphas do not line up according to ability. They do for Managers 6-9 but

[14]Technically, these results are simply the results of the last year of the 40-year simulation. The use of this last year was chosen before the simulation was run.

Manager 10 has a higher estimated alpha than Manager 9 in spite of his accuracy being lower by several percent.

VI. Correcting the Problem

Two widely used metrics for measuring two-fund asset allocation ability are the Henriksson-Merton (1981) (HM) and Treynor-Mazuy (1966) (TM) regression models. These models take the following forms,

$$R_t^e = b_0^{HM} + b_1^{HM} R_{mt}^e + b_2^{HM} (-R_{mt}^e +) + \varepsilon_t \quad (Henriksson - Merton)$$
$$R_t^e = b_0^{TM} + b_1^{TM} R_{mt}^e + b_2^{TM} (R_{mt}^e)^2 + \varepsilon_t \quad (Treynor - Mazuy)$$

where R_t^e is the portfolio return minus the risk-free rate, R_{mt}^e is the market return minus the risk-free rate, and $-R_{mt}^e + = \max(-R_{mt}^e, 0)$. The HM model works by regressing the portfolio excess return on the market excess return and the greater of either zero or the negative of the market excess return. Thus, when the market outperforms the risk-free asset, the latter variable takes on a value of zero. When the risk-free rate outperforms the market, the latter variable takes on a value of minus the market excess return. This specification attempts to model the manager who is in the market when it is outperforming the risk-free asset and out of the market in the opposite situation. The TM model works by regressing the portfolio excess return on the market excess return and the square of the market excess return. This specification attempts to detect whether the manager increases his beta as the market performs better. The coefficients b_2^{HM} and b_2^{TM} will be significantly positive if the manager has ability.

The HM and TM regressions have been used extensively in evaluating mutual funds and other professional asset allocators, but application of these models to real-world data are joint tests of the return-generating process and asset allocation ability. The hypothetical asset allocators we employed in our simulations will be particularly useful in eliminating this problem. By simulating returns that are driven only by the market, we know that we have specified the correct return-generating process. In particular, the random asset allocators will provide a valuable benchmark for the reliability of the HM and TM tests. If HM and TM are dependable methodologies, we should see few if any Type I errors, because we know that the random asset allocators have no ability. The managers imbued with varying degrees of predictive ability can also be useful to examine the HM and TM models. We know that their HM and TM measures should align with their respective proportion of markets correctly called. These clean simulated tests provide valuable benchmarks to see if the tests are reliable.

The results for a simulation of 1,000,000 runs of 250 days with an expected return of 12%, a volatility of 20%, and a risk-free rate of 4% are presented in Table 8. Panel A shows the results for the managers who have ability as specified by the percentage of times they choose the correct asset class. As we see, these results are encouraging. They line up monotonically by ability. Note the large jump between 50% and 60%. This key point corresponds to the probability that the index beats the risk-free rate and, thus, is a type of buy-and-hold benchmark. Panel B shows the results for the managers who choose their positions randomly. Unfortunately, these results are less encouraging for the HM and TM metrics. Under the null hypothesis that these managers have no ability, we find that the null is rejected anywhere from 7.3% to 9.7%, figures that can be considered Type I errors. Typically, the significance of a regression coefficient at the 5% represents a 5% chance that the independent variable is not significantly related to the dependent variable. In this context, significance means that there is only a 5% chance that we would erroneously conclude that such a manager has ability. Yet, we know that none of these managers have ability. By comparison, we found that alpha, which has no pre-designed ability to detect asset allocation skill, has no Type I error in large samples such as this.

VI. Conclusions

These results lead to several conclusions that have important implications for performance measurement of asset allocators. In a large sample, the estimated alpha seems capable of distinguishing asset allocation ability. It both recognizes ability and seems to produce zero estimated alphas for managers whose positions are selected randomly. But in practice, we do not have the luxury of such large samples. With samples of a lifetime of a manager, we can apparently identify quality managers, but we cannot eliminate the positive estimated alphas bias for hypothetical managers who select their positions using random generators. Hence, managers with no ability can be perceived as having ability. In a simulation of a single year of daily data, mangers with no ability can easily be found to show ability. Moreover, managers with different degrees of ability can be rank ordered incorrectly. This problem becomes all the more severe when we consider that actual performance evaluation is not typically done using daily data over a single year. In fact, the sample sizes are often even smaller, such as quarterly data over a year. Widely-used alternatives such as the Henriksson-Merton and Treynor-Mazuy, which are designed to measure asset allocation and not security selection, fare no better. In large samples they are unable to eliminate the Type I error that alpha, which is not designed to measure asset allocation ability, is able to do. In fact, their true Type I errors are larger than their Type I errors as implied by statistical theory.

Of course, our analysis is done only using two asset classes. In practice, many managers use more than two asset classes. Some managers might follow a strategy of attempting to indentify the best performing asset class and putting all of the portfolio in that class. Other managers might choose to allocate funds in accordance with relative class performance. The most funds would go into the best performing class and the least would go into the worst performing class. In either case, the problems identified here could not possibly go away and would likely be even more difficult to resolve, given the greater number of choices.

Thus, very good managers may be identifiable, but the estimated alpha does not discriminate well among differences in ability with reasonable sample sizes. Moreover, the estimated alpha can easily attribute quality to managers who choose their positions randomly. Given these results, it is not surprising that asset allocators boast about their "positive alphas." As we show here, a positive market risk premium and a random number generator can produce such a result and not merely be the case of random noise.

Appendix

Examination of the Relationship between the $\hat{\beta}$ and the Exposure (percentage of periods in the market) for a Two-Fund Asset Allocator

Having divided the observation period into two parts, the period in which the manager is in the market (the exposure period) and the period in which the manager is out of the market (the hedge period), the estimated beta can be expressed as[15]

$$\hat{\beta} = \frac{\sum_{t=1}^{T_i}(R_{mt}(i) - \bar{R})(R_{mt}(i) - \bar{R}_m) + \sum_{t=1}^{T_o}(r - \bar{R})(R_{mt}(o) - \bar{R}_m)}{\sum_{t=1}^{T_i}(R_{mt}(i) - \bar{R}_m)^2 + \sum_{t=1}^{T_o}(R_{mt}(o) - \bar{R}_m)^2}. \quad \text{(A.1)}$$

To simplify this expression, start with the numerator,

$$\sum_{t=1}^{T_i}(R_{mt}(i) - \bar{R})(R_{mt}(i) - \bar{R}_m) + \sum_{t=1}^{T_o}(r - \bar{R})(R_{mt}(o) - \bar{R}_m). \quad \text{(A.2)}$$

Let us focus on the first term,

$$\sum_{t=1}^{T_i}(R_{mt}(i) - \bar{R})(R_{mt}(i) - \bar{R}_m), \quad \text{(A.3)}$$

which can be further simplified. Note that the manager's average return can be expressed as a weighted average of the market return during the exposure period and the risk-free rate during the hedge period,

$$\bar{R} = p\bar{R}_m(i) + (1-p)r. \quad \text{(A.4)}$$

The average market return can be expressed as a weighted average of the average market return during the exposure period and the average market return during the hedge period,

$$\bar{R}_m = p\bar{R}_m(i) + (1-p)\bar{R}_m(o). \quad \text{(A.5)}$$

Thus, from Equation (A.5),

$$p\bar{R}_m(i) = \bar{R}_m - (1-p)\bar{R}_m(o).$$

[15] Technically, both numerator and denominator are divided by T-1.

Substituting this result into Equation (A.4), the manager's average return can be written as

$$\begin{aligned}\bar{R} &= \bar{R}_m - (1-p)\bar{R}_m(o) + (1-p)r \\ &= \bar{R}_m - (1-p)(\bar{R}_m(o) - r).\end{aligned} \tag{A.6}$$

Substituting this result into the first term in the product in Equation (A.3), we obtain

$$R_{mt}(i) - \bar{R} = R_{mt}(i) - \bar{R}_m + (1-p)(\bar{R}_m(o) - r).$$

Then Equation (A.3) becomes

$$\begin{aligned}&\sum_{t=1}^{T_i}(R_{mt}(i) - \bar{R}_m + (1-p)(\bar{R}_m(o) - r))(R_{mt}(i) - \bar{R}_m) \\ &= \sum_{t=1}^{T_i}(R_{mt}(i) - \bar{R}_m)^2 + (1-p)(\bar{R}_m(o) - r))\sum_{t=1}^{T_i}(R_{mt}(i) - \bar{R}_m)\end{aligned} \tag{A.7}$$

Now let us focus on the second term in the numerator of Equation (A.1),

$$\sum_{t=1}^{T_o}(r - \bar{R})(R_{mt}(o) - \bar{R}_m). \tag{A.8}$$

Let us define a new term, $w_t(o)$ to represent the difference at time t between the market return and the risk-free rate,

$$w_t(o) = R_{mt}(o) - r. \tag{A.9}$$

Then, the risk-free rate is

$$r = R_{mt}(o) - w_t(o). \tag{A.10}$$

Using this result and Equation (A.6), the first term in the product in Equation (A.8) is

$$r - \bar{R} = (R_{mt}(o) - \bar{R}_m) + (1-p)(\bar{R}_m(o) - r) - w_t(o). \tag{A.11}$$

Thus, Equation (A.8) becomes

$$\begin{aligned}&\sum_{t=1}^{T_o}\big((R_{mt}(o) - \bar{R}_m) + (1-p)(\bar{R}_m(o) - r) - w_t(o)\big)(R_{mt}(o) - \bar{R}_m) \\ &= \sum_{t=1}^{T_o}(R_{mt}(o) - \bar{R}_m)^2 + (1-p)(\bar{R}_m(o) - r)\sum_{t=1}^{T_o}(R_{mt}(o) - \bar{R}_m) - \sum_{t=1}^{T_0} w_t(o)(R_{mt}(o) - \bar{R}_m).\end{aligned} \tag{A.12}$$

Now let us return to beta. From the above results, the estimated beta will be

$$\hat{\beta} = \frac{\sum_{t=1}^{T_i}(R_{mt}(i)-\bar{R}_m)^2 + \sum_{t=1}^{T_o}(R_{mt}(o)-\bar{R}_m)^2}{\sum_{t=1}^{T_i}(R_{mt}(i)-\bar{R}_m)^2 + \sum_{t=1}^{T_o}(R_{mt}(o)-\bar{R}_m)^2}$$
$$+\frac{(1-p)(\bar{R}_m(o)-r))\sum_{t=1}^{T_i}(R_{mt}(i)-\bar{R}_m)+(1-p)(\bar{R}_m(o)-r)\sum_{t=1}^{T_o}(R_{mt}(o)-\bar{R}_m)-\sum_{t=1}^{T_0}w_t(o)(R_{mt}(o)-\bar{R}_m)}{\sum_{t=1}^{T_i}(R_{mt}(i)-\bar{R}_m)^2 + \sum_{t=1}^{T_o}(R_{mt}(o)-\bar{R}_m)^2}. \quad \text{(A.13)}$$

Let us define the term following the plus sign as gamma, where

$$\gamma = \frac{(1-p)(\bar{R}_m(o)-r))\sum_{t=1}^{T_i}(R_{mt}(i)-\bar{R}_m)+(1-p)(\bar{R}_m(o)-r)\sum_{t=1}^{T_o}(R_{mt}(o)-\bar{R}_m)-\sum_{t=1}^{T_0}w_t(o)(R_{mt}(o)-\bar{R}_m)}{\sum_{t=1}^{T_i}(R_{mt}(i)-\bar{R}_m)^2 + \sum_{t=1}^{T_o}(R_{mt}(o)-\bar{R}_m)^2}. \quad \text{(A.14)}$$

We can simplify this expression. In the numerator, in the first summation, we have the deviations from the overall market mean during the exposure period. In the second summation, we have the deviations from the overall mean during the hedge period. These two expressions combine to equal the sum of the deviations from the overall mean,

$$\gamma = \frac{(1-p)(\bar{R}_m(o)-r))\sum_{t=1}^{T_i}(R_{mt}(i)-\bar{R}_m)+(1-p)(\bar{R}_m(o)-r)\sum_{t=1}^{T_o}(R_{mt}(o)-\bar{R}_m)-\sum_{t=1}^{T_0}w_t(o)(R_{mt}(o)-\bar{R}_m)}{\sum_{t=1}^{T_i}(R_{mt}(i)-\bar{R}_m)^2 + \sum_{t=1}^{T_o}(R_{mt}(o)-\bar{R}_m)^2}$$
$$= \frac{(1-p)(\bar{R}_m(o)-r))\sum_{t=1}^{T}(R_{mt}-\bar{R}_m)-\sum_{t=1}^{T_0}w_t(o)(R_{mt}(o)-\bar{R}_m)}{\sum_{t=1}^{T_i}(R_{mt}(i)-\bar{R}_m)^2 + \sum_{t=1}^{T_o}(R_{mt}(o)-\bar{R}_m)^2}.$$

Of course, the sum of the deviations from the overall mean is zero. Thus,

$$\gamma = \frac{-\sum_{t=1}^{T_0}w_t(o)(R_{mt}(o)-\bar{R}_m)}{\sum_{t=1}^{T_i}(R_{mt}(i)-\bar{R}_m)^2 + \sum_{t=1}^{T_o}(R_{mt}(o)-\bar{R}_m)^2}.$$

At this point, it will be helpful to return to the original definition of $w_t(o)$, Equation (A.9). Thus, gamma becomes

$$\gamma = \frac{-\sum_{t=1}^{T_0}(R_{mt}(o)-r)(R_{mt}(o)-\bar{R}_m)}{\sum_{t=1}^{T_i}(R_{mt}(i)-\bar{R}_m)^2+\sum_{t=1}^{T_o}(R_{mt}(o)-\bar{R}_m)^2} \tag{A.15}$$

And the estimated beta is 1 – γ where γ is given by (A.15). The "1" represents the estimated beta during the exposure period. The γ is an adjustment for the period when the manager is hedged. In examining γ we see that the numerator is the sum of the product of two terms. The first is the amount gained by the manager by being in the risk-free asset relative to being in the market. The second is the difference between what the market returns during the exposure period and the overall average market return. Thus, this term is a type of covariance that reduces the manager's estimated beta during the hedge period. We should not think of a manager's estimated beta during the hedge period as zero but more like the negative of the relationship between what he gives up and how the market performs during the hedge period.

To analyze γ, let us proceed in the following manner. Focusing on the numerator, we can add and subtract the average market return to obtain

$$\begin{aligned}
&-\sum_{t=1}^{T_0}(R_{mt}(o)-r)(R_{mt}(o)-\bar{R}_m)\\
&=-\sum_{t=1}^{T_0}(R_{mt}(o)-r-\bar{R}_m+\bar{R}_m)(R_{mt}(o)-\bar{R}_m)\\
&=-\sum_{t=1}^{T_0}(R_{mt}(o)-\bar{R}_m+\bar{R}_m-r)(R_{mt}(o)-\bar{R}_m)\\
&=-\sum_{t=1}^{T_0}(R_{mt}(o)-\bar{R}_m)^2-(\bar{R}_m-r)\sum_{t=0}^{T_o}(R_{mt}(o)-\bar{R}_m).
\end{aligned}$$

This makes γ be

$$\begin{aligned}
\gamma &= \frac{-\sum_{t=1}^{T_0}(R_{mt}(o)-\bar{R}_m)^2-(\bar{R}_m-r)\sum_{t=0}^{T_o}(R_{mt}(o)-\bar{R}_m)}{\sum_{t=1}^{T_i}(R_{mt}(i)-\bar{R}_m)^2+\sum_{t=1}^{T_o}(R_{mt}(o)-\bar{R}_m)^2}\\
&=\frac{-\sum_{t=1}^{T_0}(R_{mt}(o)-\bar{R}_m)^2}{\sum_{t=1}^{T_i}(R_{mt}(i)-\bar{R}_m)^2+\sum_{t=1}^{T_o}(R_{mt}(o)-\bar{R}_m)^2}+\frac{-(\bar{R}_m-r)\sum_{t=0}^{T_o}(R_{mt}(o)-\bar{R}_m)}{\sum_{t=1}^{T_i}(R_{mt}(i)-\bar{R}_m)^2+\sum_{t=1}^{T_o}(R_{mt}(o)-\bar{R}_m)^2}.
\end{aligned} \tag{A.16}$$

The first ratio on the right-hand side is the sum of squares of the market return during the hedge period divided by the total sum of squares of the market return. We conjecture that this ratio, ignoring the negative sign, will be approximately 1 - p. Let us define it as 1 - p* and examine its value. It can be expressed as follows:

$$
\begin{aligned}
1 - p^* &= \frac{\sum_{t=1}^{T_0} (R_{mt}(o) - \bar{R}_m)^2}{\sum_{t=1}^{T_i} (R_{mt}(i) - \bar{R}_m)^2 + \sum_{t=1}^{T_o} (R_{mt}(o) - \bar{R}_m)^2} \\
&= \frac{\left(\sum_{t=1}^{T_0} (R_{mt}(o) - \bar{R}_m)^2 \right) / (T-1)}{\left(\sum_{t=1}^{T_i} (R_{mt}(i) - \bar{R}_m)^2 \right) / (T-1) + \left(\sum_{t=1}^{T_o} (R_{mt}(o) - \bar{R}_m)^2 \right) / (T-1)}.
\end{aligned}
\tag{A.17}
$$

Define the two sub-period variances as

$$
\begin{aligned}
\sigma_m{}^2(i) &= \left(\sum_{t=1}^{T_i} (R_{mt}(i) - \bar{R}_m)^2 \right) / (T_i - 1) \\
\sigma_m{}^2(o) &= \left(\sum_{t=1}^{T_o} (R_{mt}(o) - \bar{R}_m)^2 \right) / (T_o - 1).
\end{aligned}
\tag{A.18}
$$

Note that in estimating these variances, we subtract the overall mean, not the respective sub-period means. Substituting these results into (A.17), we obtain

$$
1 - p^* = \frac{\sigma_m{}^2(o)\left(\frac{T_o - 1}{T-1}\right)}{\sigma_m{}^2(i)\left(\frac{T_i - 1}{T-1}\right) + \sigma_m{}^2(o)\left(\frac{T_o - 1}{T-1}\right)}.
$$

In the absence of volatility timing, the sub-period variances should be equal. Thus, let us establish that these variances equal a common value,

$$
\sigma_m{}^2(i) = \sigma_m{}^2(i) = \sigma_m{}^2(*).
$$

Substituting back, we can now reduce 1 – p* to a very simple expression,

$$1-p^* = \frac{\sigma_m{}^2(*)\left(\frac{T_o-1}{T-1}\right)}{\sigma_m{}^2(*)\left(\frac{T_i-1}{T-1}\right)+\sigma_m{}^2(*)\left(\frac{T_o-1}{T-1}\right)}$$
$$= \frac{\left(\frac{T_o-1}{T-1}\right)}{\left(\frac{T_i-1}{T-1}\right)+\left(\frac{T_o-1}{T-1}\right)} = \frac{T_o-1}{T_i-1+T_o-1}$$
$$= \frac{T_o-1}{T-2}.$$

It follows that

$$p^* = \frac{T_i-1}{T-2}.$$

With $p = T_i/T$, the ratio p^* does not appear to equal p in finite samples. It will in one special case, $p = ½$, but more generally it will not. Examinations of its properties reveal that the difference between p^* and p will shrink with the sample size toward a limit of zero with an infinite sample. More generally, when $p <$ ($>$) ½, $p^* >$ ($<$) p.

Now we have γ as

$$\gamma = -(1-p^*) + \frac{-(\bar{R}_m - r)\sum_{t=0}^{T_o}(R_{mt}(o)-\bar{R}_m)}{\sum_{t=1}^{T_i}(R_{mt}(i)-\bar{R}_m)^2+\sum_{t=1}^{T_o}(R_{mt}(o)-\bar{R}_m)^2}.$$

Examining the term in the numerator, we have

$$-(\bar{R}_m - r)\sum_{t=0}^{T_o}(R_{mt}(o)-\bar{R}_m) = -(\bar{R}_m - r)\left(\sum_{t=0}^{T_o}R_{mt}(o)-T_o\bar{R}_m\right)$$
$$= -(\bar{R}_m - r)(T_o\bar{R}_m(o)-T_o\bar{R}_m)$$
$$= -(\bar{R}_m - r)T_o(\bar{R}_m(o)-\bar{R}_m).$$

Substituting from Equation (A.5), we obtain

$$-(\bar{R}_m - r)T_o(\bar{R}_m(o) - p\bar{R}_m(i) - (1-p)\bar{R}_m(o))$$
$$= -(\bar{R}_m - r)T_o p(\bar{R}_m(o)-\bar{R}_m(i)).$$

Thus, γ is

$$\gamma = -(1-p^*) + \frac{-(\bar{R}_m - r)T_o p(\bar{R}_m(o) - \bar{R}_m(i))}{\sum_{t=1}^{T_i}(R_{mt}(i) - \bar{R}_m)^2 + \sum_{t=1}^{T_o}(R_{mt}(o) - \bar{R}_m)^2}.$$

And β is

$$\begin{aligned}\hat{\beta} &= 1-(1-p^*) + \frac{-(\bar{R}_m - r)T_o p(\bar{R}_m(o) - \bar{R}_m(i))}{\sum_{t=1}^{T_i}(R_{mt}(i) - \bar{R}_m)^2 + \sum_{t=1}^{T_o}(R_{mt}(o) - \bar{R}_m)^2} \\ &= p^* - \frac{(\bar{R}_m - r)T_o p(\bar{R}_m(o) - \bar{R}_m(i))}{\sum_{t=1}^{T_i}(R_{mt}(i) - \bar{R}_m)^2 + \sum_{t=1}^{T_o}(R_{mt}(o) - \bar{R}_m)^2}.\end{aligned} \tag{A.19}$$

Thus, we see that the estimated beta will be p^* plus the ratio on the right-hand side. As noted earlier, p^* will not equal p in finite samples and in the special case of $p = ½$, though it may be close quite generally. We see that this ratio will be driven by the ex post market risk premium, the exposure, and the difference in the market return when the manager is in and out of the market, which reflects the manager's ability. It can be shown that p^* relates to p in the following manner:

$$p^* = \frac{Tp-1}{T-2}.$$

Hence, $\hat{\beta}$ relates to p as follows:

$$\hat{\beta} = \frac{Tp-1}{T-2} - \frac{(\bar{R}_m - r)T_o p(\bar{R}_m(o) - \bar{R}_m(i))}{\sum_{t=1}^{T_i}(R_{mt}(i) - \bar{R}_m)^2 + \sum_{t=1}^{T_o}(R_{mt}(o) - \bar{R}_m)^2}. \tag{A.20}$$

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Figure 1
Value of $1 invested in a market index over two periods with a probability of an up move in each period of 0.65. The market can move up 20% or down 10% in the first period and up 25% or down 20% in the second period.

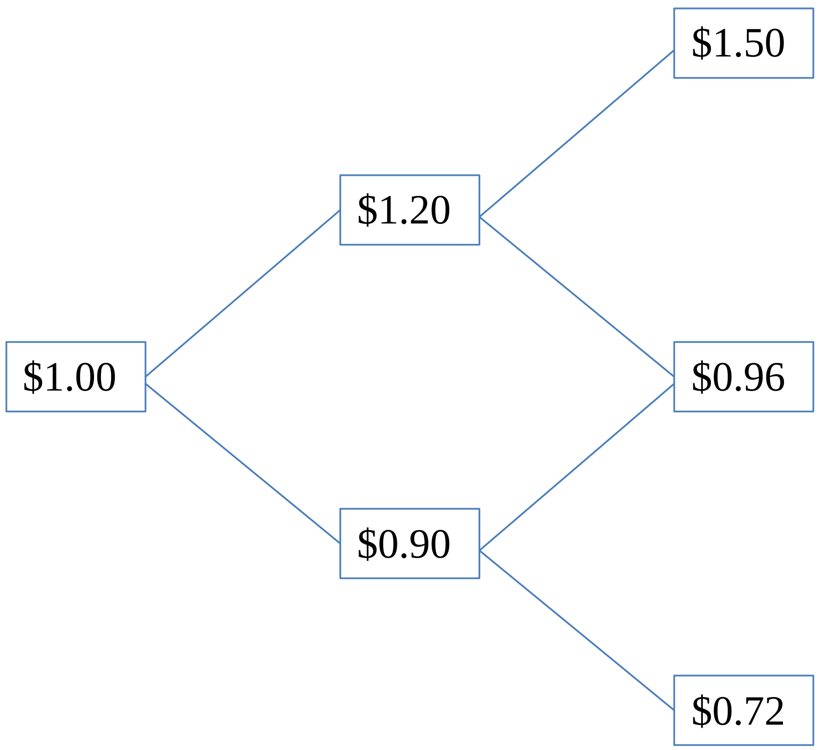


Figure 2

Value of $1 invested by each manager (1-4) over two binomial periods with a probability of an up move in each period of 0.65. The market can move up 20% or down 10% in the first period and up 25% or down 20% in the second period.

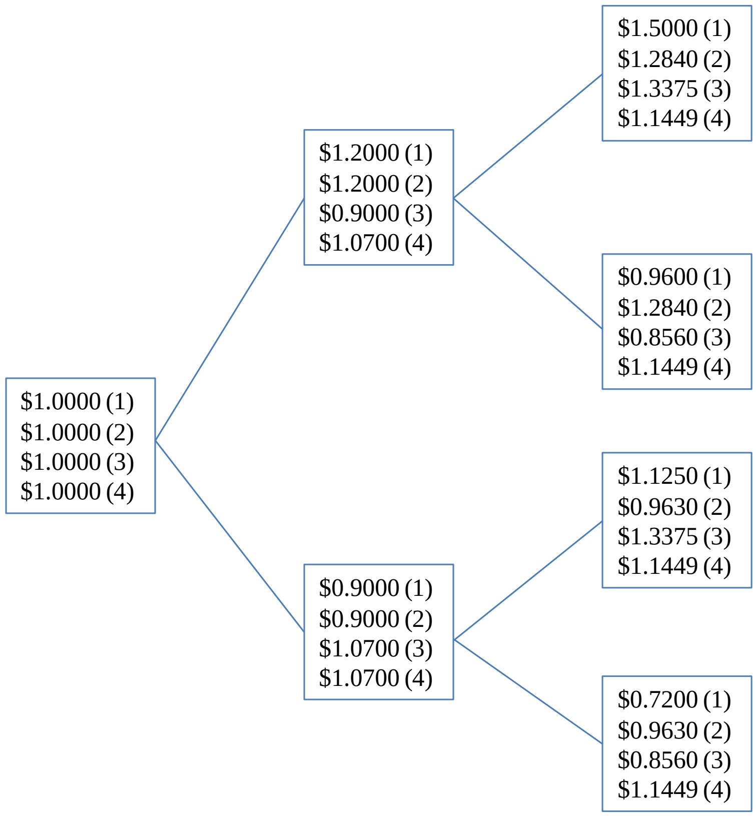


Figure 3

The difference between *p*, the exposure, and $\hat{\beta}$, for various exposures and sample sizes for managers with no asset allocation ability. Asset allocation ability is indicated by no difference between the average market return while the manager is in the market and the average market return while the manager is out of the market. Exposure is the percentage of periods the manager is in the market. The trading period is a day. Sample sizes of 100, 250, 1000, 5000, 10000, and 100000 are indicated.

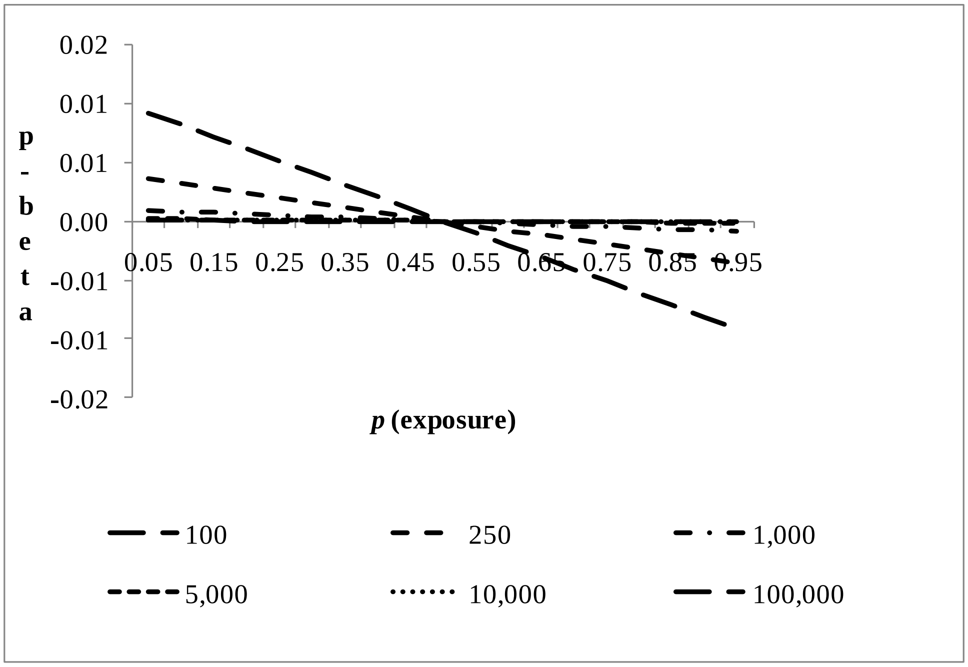
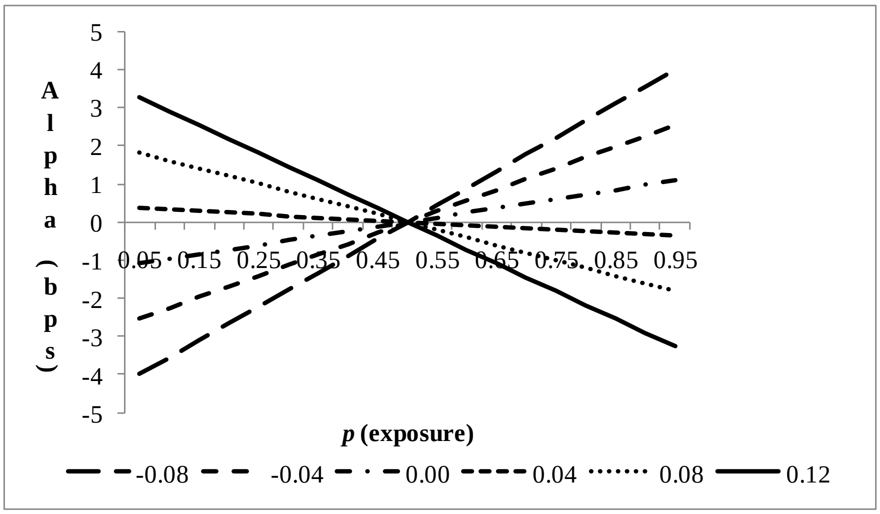


Figure 4

Relationship between the estimated alpha (in basis points) and the manager's exposure (*p*) by the level (average return) of the market and sample size (number of trading periods) for managers with no asset allocation ability. Asset allocation ability is indicated by the difference between the average market return while the manager is in the market and the average market return while the manager is out of the market. The volatility of the market is 20%, and the risk-free rate is 3%. Exposure is the percentage of periods the manager is in the market. The trading period is a day.

(a) By level (average return) of the market. Sample size (number of trading periods) is 250. Average market returns of -8%, -4%, 0%, 4%, 8%, and 12% are indicated.



(b) By sample size (number of trading periods). Sample sizes of 100, 250, 1000, 5000, 10000, and 100000 are indicated.

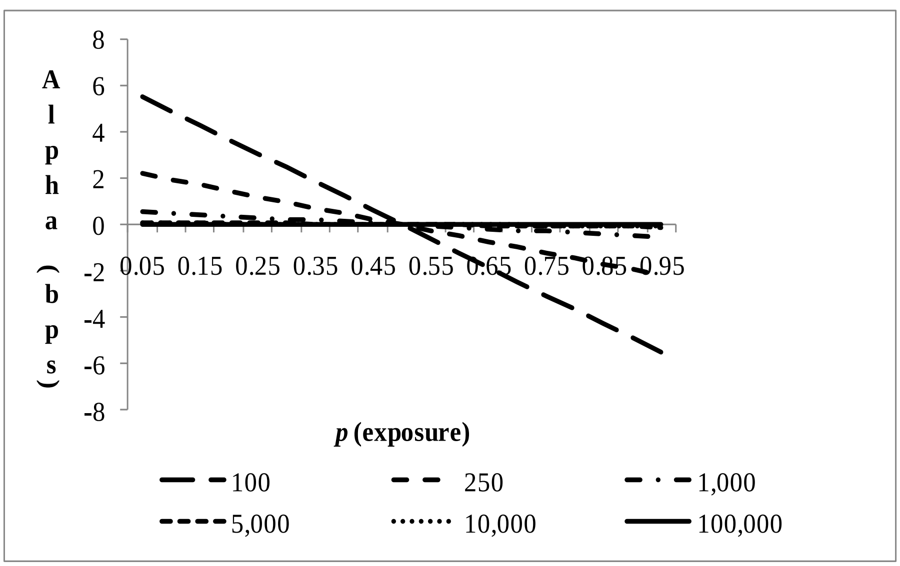
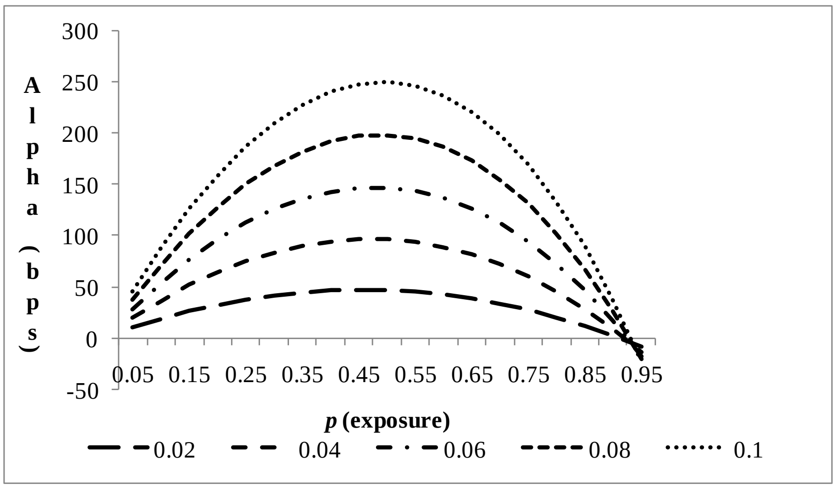


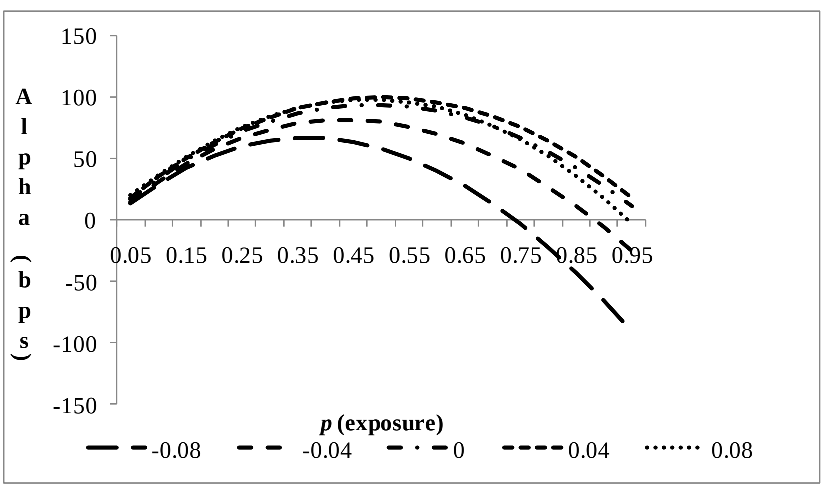
Figure 5

Relationship between the estimated alpha (in basis points) and the manager's exposure by the level (average return) of the market and sample size (number of trading periods) for managers with asset allocation ability. Asset allocation ability is defined by a positive difference between the average market return while the manager is in the market and the average market return while the manager is out of the market. The risk-free rate is 3%. Exposure is the percentage of periods the manager is in the market. The trading periods is a day.

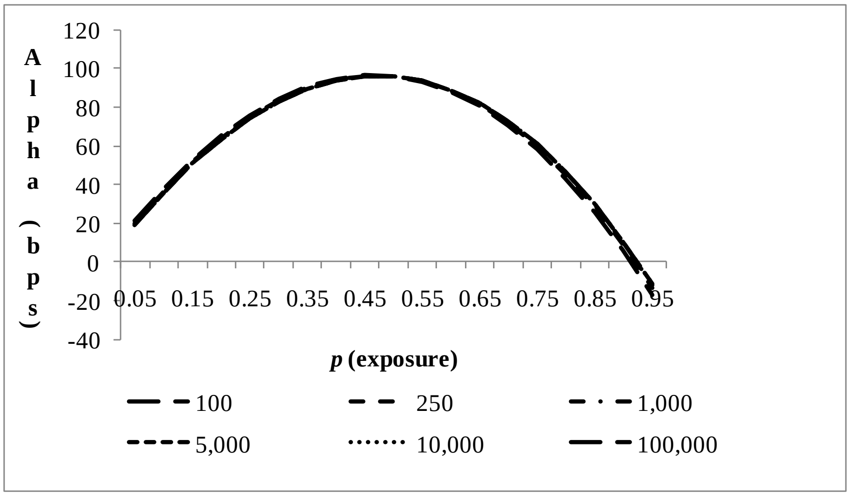
(a) By different values of the difference between average market return while manager is in and out of the market. Average market return is 9%, market volatility is 20%, and sample size (number of trading periods) is 250. Difference between average market return while in and out of the market is given as 2%, 4%, 6%, 8%, and 10% are indicated.



(b) By level (average return) of the market. Ability is 4%, market volatility is 20%, and sample size (number of trading periods) is 250. Average market returns of -8%, -4%, 0%, 4%, 8%, and 12% are indicated.



(c) By sample size (number of trading periods). Level (average return) of the market is 9%, market volatility is 20%, and ability is 4%. Sample sizes of 100, 250, 1000, 5000, 10000, and 100000 are indicated.



(d) By volatility. Level (average return) of the market is 9%, ability is 4%, and the number of trading periods is 250. Volatilities of 10%, 20%, 30%, 40%, and 50% are indicated.

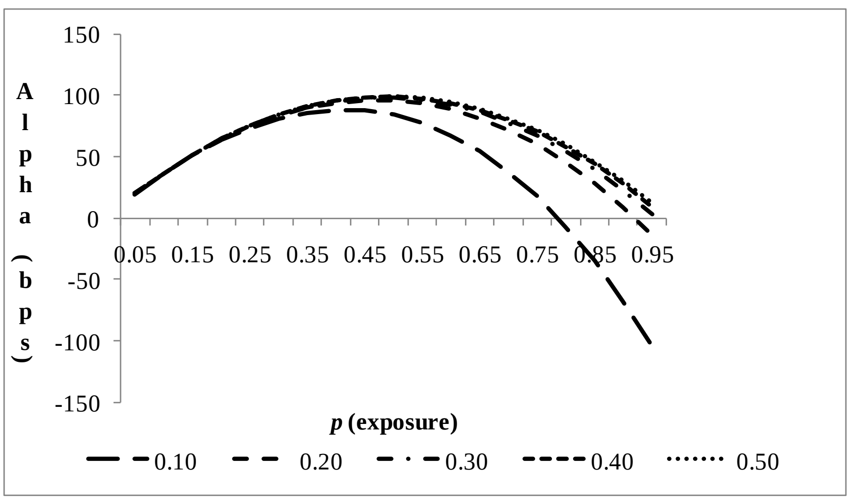


Table 1
Estimated alphas for each of the four possible outcomes for four managers engaged in asset allocation strategies over two binomial periods with a probability of an up move in each period of 0.65. The market can move up 20% or down 10% in the first period and up 25% or down 20% in the second period. The risk-free rate is 3%.

	Outcome			
Manager	1	2	3	4
1	0.0000	0.0000	0.0000	0.0000
2	0.7480	0.0978	-0.0817	-0.2990
3	-0.7480	-0.0978	0.0817	0.2990
4	0.0000	0.0000	0.0000	0.0000

Table 2
The Proportion of Correct Market Calls for a Manager with No Ability, Given Various Probabilities of a Good Market (Stocks beat Cash).

	Probability that Manager is In the Market				
Probability of Good Market	0.25	0.40	0.50	0.60	0.75
0.40	0.55	0.52	0.50	0.48	0.45
0.42	0.54	0.52	0.50	0.48	0.46
0.44	0.53	0.51	0.50	0.49	0.47
0.46	0.52	0.51	0.50	0.49	0.48
0.48	0.51	0.50	0.50	0.50	0.49
0.50	0.50	0.50	0.50	0.50	0.50
0.52	0.49	0.50	0.50	0.50	0.51
0.54	0.48	0.49	0.50	0.51	0.52
0.56	0.47	0.49	0.50	0.51	0.53
0.58	0.46	0.48	0.50	0.52	0.54
0.60	0.45	0.48	0.50	0.52	0.55

Table 3
The Implied Exposure of a Manager with Accuracy, Given Various Ranges of Accuracy

	Probability that Manager is In Correct Asset Class (p_c = accuracy)				
Probability of Good Market	0.25	0.40	0.50	0.60	0.75
0.40	0.55	0.52	0.50	0.48	0.45
0.42	0.54	0.52	0.50	0.48	0.46
0.44	0.53	0.51	0.50	0.49	0.47
0.46	0.52	0.51	0.50	0.49	0.48
0.48	0.51	0.50	0.50	0.50	0.49
0.50	0.50	0.50	0.50	0.50	0.50
0.52	0.49	0.50	0.50	0.50	0.51
0.54	0.48	0.49	0.50	0.51	0.52
0.56	0.47	0.49	0.50	0.51	0.53
0.58	0.46	0.48	0.50	0.52	0.54
0.60	0.45	0.48	0.50	0.52	0.55

Table 4
Characteristics of 12 hypothetical asset allocators used in simulation tests.

Manager #	Characteristic
1	Always invested in market index
2	In market index 75% of days, in risk-free asset 25% of days, with position determined by random number generator
3	In market index 50% of days, in risk-free asset 50% of days, with position determined by random number generator
4	In market index 25% of days, in risk-free 75% of days, with position determined by random number generator
5	Always invested in risk-free asset
6	Always invested in the higher performing asset class (perfect foresight)
7	Invested in the higher performing asset class 70% of days and the lower performing asset class 30% of days
8	Invested in the higher performing asset class 65% of days and the lower performing asset class 35% of days
9	Invested in the higher performing asset class 60% of days and the lower performing asset class 40% of days
10	Invested in the higher performing asset class 55% of days and the lower performing asset class 45% of days
11	Invested in the higher performing asset class 50% of days and the lower performing asset class 50% of days
12	Invested in the higher performing asset class 45% of days and the lower performing asset class 55% of days

Table 5

Results of Monte Carlo simulation with 1,000,000 runs of 250-day periods. There are two asset classes, a market index and the risk-free asset. The expected return on the market index is 12% and the volatility of the market index is 20%. The risk-free rate is 5%. The market return is generated by assuming that it follows a geometric Brownian motion process. A description of each manager is provided in Table 2. Exposure (%) is the number of days in each 250-day period that the manager is in the market divided by 250. Accuracy (%) is the number of days the asset class the manager is in during the 250-day period outperformed the other asset class divided by 250. The next columns contain the manager's average return, volatility, and estimated beta over each 250-day period, respectively. The next column is the difference between the average market return during the periods in which the manager is in and out of the market. The final column is the average estimated alpha over each 250-day period. All figures below are averages across the entire 1,000,000 runs.

Manager	**Exposure (%)**	**Accuracy (%)**	$\bar{R}(\%)$	$\sigma(\%)$	$\hat{\beta}$	$\bar{R}(i) - \bar{R}(o)$	$\hat{\alpha}(\%)$
1	100.00	50.80	12.00	19.98	1.00	NA	0.00
2	75.00	50.42	10.25	17.30	0.75	0.00	-0.00
3	50.00	50.00	8.50	14.11	0.50	0.00	-0.01
4	25.00	49.57	6.75	9.95	0.25	0.00	-0.00
5	0.00	49.14	5.00	0.00	0.00	NA	-0.00
6	50.86	100.00	135.22	11.78	0.51	506.79	126.16
7	50.32	70.00	59.13	13.80	0.50	202.44	50.41
8	50.26	65.00	46.52	13.94	0.50	152.03	37.85
9	50.16	60.00	33.78	14.04	0.50	101.09	25.17
10	50.10	55.00	21.16	14.10	0.50	50.61	12.60
11	50.00	50.00	8.46	14.11	0.50	-0.20	-0.05
12	49.92	45.00	-4.17	14.07	0.50	-50.68	-12.17

Table 6

Results of Monte Carlo simulation with 40 runs of 250-day periods. There are two asset classes, a market index and the risk-free asset. The expected return on the market index is 12% and the volatility of the market index is 20%. The risk-free rate is 5%. The market return is generated by assuming that it follows a geometric Brownian motion process. A description of each manager is provided in Table 2. Exposure (%) is the number of days in each 250-day period that the manager is in the market divided by 250. Accuracy (%) is the number of days the asset class the manager is in during the 250-day period outperformed the other asset class divided by 250. The next columns contain the manager's average return, volatility, and estimated beta over each 250-day period, respectively. The next column is the difference between the average market return during the periods in which the manager is in and out of the market. The final column is the average estimated alpha over each 250-day period. All figures below are averages across the entire 40 runs.

Manager	**Exposure (%)**	**Accuracy (%)**	$\bar{R}$(%)	σ(%)	$\hat{\beta}$	$\bar{R}(i) - \bar{R}(o)$	$\hat{\alpha}$(%)
1	100.00	51.20	15.14	19.81	1.00	NA	0.00
2	75.20	50.88	12.91	17.20	0.75	0.27	0.11
3	49.79	50.97	12.65	13.90	0.49	10.73	2.58
4	24.88	49.66	9.11	9.77	0.24	8.02	1.62
5	0.00	48.80	5.00	0.00	0.00	NA	0.00
6	51.20	100.00	135.80	11.72	0.51	502.31	125.15
7	50.22	70.76	61.86	13.72	0.51	205.96	51.27
8	50.81	65.15	47.74	13.93	0.51	151.13	37.86
9	49.79	60.77	39.43	13.84	0.50	117.46	29.41
10	50.36	54.64	21.69	14.06	0.51	46.34	11.56
11	49.70	50.22	11.50	14.02	0.50	5.05	1.20
12	49.34	45.22	0.66	13.90	0.50	-38.22	-9.34

Table 7

Results of the final Monte Carlo simulation run for the 40-year period (see Table 4). There are two asset classes, a market index and the risk-free asset. The expected return on the market index is 12% and the volatility of the market index is 20%. The risk-free rate is 5%. The market return is generated by assuming that it follows a geometric Brownian motion process. A description of each manager is provided in Table 2. Exposure (%) is the number of days in each 250-day period that the manager is in the market divided by 250. Accuracy (%) is the number of days the asset class the manager is in during the 250-day period outperformed the other asset class divided by 250. The next columns contain the manager's average return, volatility, and estimated beta over each 250-day period, respectively. The next column is the difference between the average market return during the periods in which the manager is in and out of the market. The final column is the average estimated alpha over each 250-day period.

Manager	**Exposure (%)**	**Accuracy (%)**	$\bar{R}$(%)	σ(%)	$\hat{\beta}$	$\bar{R}(i)$ - $\bar{R}$(o)	$\hat{\alpha}$(%)
1	100.00	52.00	15.21	19.99	1.00	15.21	0.00
2	75.20	52.80	12.99	17.57	0.77	1.65	0.10
3	50.40	47.20	1.43	13.16	0.43	-34.87	-8.00
4	28.80	51.20	15.71	9.79	0.24	37.89	8.26
5	0.00	48.00	5.00	0.00	0.00	-15.21	0.00
6	52.00	100.00	136.37	11.41	0.49	505.04	126.41
7	56.40	71.60	65.02	13.77	0.50	220.65	54.86
8	53.20	66.00	52.64	14.78	0.56	169.51	41.87
9	54.00	62.00	39.43	12.95	0.43	114.40	30.05
10	53.60	60.80	46.19	13.92	0.50	113.63	36.11
11	54.40	56.00	27.15	15.04	0.57	66.90	16.34
12	52.40	50.80	8.25	15.33	0.59	-8.43	-2.76

Table 8

Henriksson-Merton and Treynor-Mazuy Measures Applied to Hypothetical Asset Allocators with managers of varying degrees of ability (Panel A) and managers with no ability (Panel B). The managers with no ability have their positions chosen by a random number generator designed to put them in the market the percentage of periods specified as their exposure. The expected return is 12%, the volatility is 20%, and the risk-free rate is 4%. The numbers in the second and third columns are the number of times in a simulation of 1,000,000 runs of 250 days each that the HM or TM coefficient is positive and statistically significant in a one-tailed test.

A. Managers with Varying Degrees of Ability

Manager (% accuracy)	Henriksson-Merton (HM)	Treynor-Mazuy (TM)
10%	0.00%	0.00%
20%	0.00%	0.00%
30%	0.00%	0.00%
40%	0.19%	0.18%
50%	14.06%	13.90%
60%	76.39%	76.24%
70%	99.53%	99.52%
80%	100.00%	100.00%
90%	100.00%	100.00%

A. Managers with no Ability

Manager (Exposure)	Henriksson-Merton (HM)	Treynor-Mazuy (TM)
10%	9.71%	9.55%
20%	8.61%	8.51%
30%	7.34%	7.27%
40%	7.38%	7.31%
50%	8.70%	8.59%

60%	7.40%	7.33%
70%	7.46%	7.39%
80%	7.44%	7.36%
90%	7.39%	7.31%