

**THE VALUATION OF CORPORATE DIVIDENDS
AS CONTINGENT CLAIMS**

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This paper illustrates how corporate dividends can be viewed as contingent claims on the cash flow of the firm. We model a firm with random cash flows and various assumptions about the size of its capital budget, its willingness to use external financing, and any maximum or minimum dividends it sets. We show that the dividend component of the value of the stock can be analyzed using option pricing theory.

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The value of a stock is widely accepted to be the value of an infinite stream of dividends. Many stock valuation models, such as the classic dividend discount model (Gordon (1959), Williams (1938)), are simplified by decomposing the model into the value of one or more upcoming dividends with the remaining dividends subsumed within a capital gains component. This approach is more or less mandatory or it would be necessary to specify an extremely long, if not infinite, series of dividends.

There are other strong justifications for the need to decompose a stock's price into a series of specified dividends during an upcoming finite period. For example, stock valuation models that incorporate differential taxation of dividends and capital gains would require such a dichotomy. Since dividends are often viewed as a tangible signal of the views of corporate management, financial analysts and market observers are also interested in knowing how much of a stock's price is accounted for by the value of one or more upcoming dividends. Many ETF and mutual fund investment strategies are geared toward identifying high dividend-paying firms.¹ Stock indexes based on high-yielding equities, such as the STOXX high yielding index family, have begun to develop. There is clearly a significant increase in dividend-based investing in recent years.² Moreover, dividend uncertainty has apparently increased. For example, *The Wall Street Journal's* MarketBeat web site reported that in 2008, 61 of the S&P 500 firms decreased their dividend collectively by about \$41 billion. That dollar figure was then broken in just the first quarter of 2009.³

Another situation in which it would be beneficial to know the value of the dividend component is in option valuation. Options are typically priced by separating the stock price into the component that reflects the present value of the dividends over the life of the option and the remaining component, which reflects the capital gains or the dividends beyond the life of the option. Standard models assume known dividends over the life of the option, thereby ignoring the effect of dividend uncertainty.⁴ Also, in

¹For example, the Wisdom Tree ETF firm has in recent years become noted for its heavy emphasis on dividends in its investment philosophy.

²See Pengelly (2010) for a possible explanation of this reason, which relates to the sharp drop in the market in 2008 that led to a downward revision in dividend expectations. Speculators see such adjustments as opportunities. Moreover,

³See <http://blogs.wsj.com/marketbeat/2009/03/06/dividend-reductions-surpass-2008s-record/>

⁴Chance, Kumar, and Rich (2002), however, have shown that options on stocks with stochastic dividends can be valued using the standard Black-Scholes-Merton approach if the dividends can

the last 10 years, the exchange-listed and over-the-counter derivatives industry has seen the development of derivatives on dividends, such as futures, options, and swaps that pay off based on the dividend component of widely-recognized indexes.⁵

Decomposing a stock's value into dividends and capital gains is, thus, an important and necessary exercise for much of the process of financing modeling, but it is not a simple straightforward procedure. Brennan (1998) was probably the first paper to recognize the benefit of decomposing the returns on a stock index into dividends and capital gains. There has been a tremendous surge in recent years in research on dividend modeling and risk, e. g., Binsbergen et al (2010), Chen, Da, and Priestley (2009), Lu and Karaban (2009), Buehler et al (2010), and Kruchen and Vanini (2008). Most of these models are based on probabilistic forecasting approaches. This paper examines dividends at a much more fundamental, corporate-based level. It takes the approach that a company has internal and possibly externally-generated financing that can be expended on either capital investment or dividends or a combination of both. It assumes that capital investment has priority over dividends, in that the former can create wealth while the latter is wealth-neutral. Depending on the specific dividend policy followed by the firm, dividends are seen as a contingent claim. That is, a dividend is like an option held by the company. Its priority is to meet its capital budget. If it does so, it can pay dividends. The dividend is thus, like an option, having a non-negative value. The non-payment of a dividend is like an option unexercised, which occurs when the available cash is insufficient to pay the dividend, given the desire to meet a capital budget. By examining the analogy of dividends to options, many of the results of standard option pricing theory can be used to provide valuable insights. Moreover, the flexibility of option theory, such as its ability to accommodate stochastic exercise prices, means that we can analyze the dividends under a variety of policies related to the payment of dividends and a planned capital budget.

In Section I, we develop the model. In Section II we analyze the model results under various scenarios related to dividends and capital budget priorities. Section III contains our conclusions.

I. Development of the Model

be valued separately. That is, as long as one can derive the present value of the (stochastic) dividends, the Black-Scholes-Merton model still holds in precisely the same format, requiring the stock price minus the present value of the dividends.

⁵As an example of an exchange-listed product based on dividends, see <http://www.cboe.com/micro/dvs/introduction.aspx>

We start with a single-period with a horizon date of time T in which cash is generated by an all-equity firm at time T in the amount C_T .⁶ This cash can then be allocated between dividends and capital investment.⁷ At time T , the firm will pay a dividend of D_T and reinvest an amount $B(T)$. The amount $B(T)$ can be viewed as the capital budget to be invested at T with the amount established at a prior date.⁸ We explore a number of scenarios in which B_T , D_T or both are fixed or deterministically variable. We later examine the possibility of a stochastic capital budget, which would reflect the randomness of the arrival of projects with economic profits.

To establish the primary result of this paper – that dividends can be valued using contingent claims theory - we do not require any statements about the probability distribution or stochastic process of cash flow. There are, however, additional conclusions we can draw if we make some assumptions about the stochastic properties of cash flow, which we shall do at the appropriate point.

We position ourselves at time t with the objective of determining the value of the dividend to be paid at time T . To assess the value at t of the dividend at T , we require that the current value of the cash flow, C_t .⁹ Although such a value is not a publicly-reported figure, it should not be a difficult value for financial analysts to estimate, given that they routinely make estimates of future cash flows. Assigning a current value to a series of future cash flows is merely applying an appropriate discount rate to a standard forecasted variable.¹⁰ We assume no taxes or transaction costs, as well as other standard perfect market assumptions. We also hold corporate investment policy constant by assuming that all investment generates no economic value, or zero net present value

⁶Cash can be generated prior to time T , but if we assume that this cash is accumulated and reinvested until time T , we would need a dynamic volatility model, since the value of cash at time T is the underlying state variable. The uncertainty of the cash at T is gradually resolved as cash is accumulated up to time T , thereby resulting in lower volatility as the horizon approaches. This effect is similar to that of a bond price as it approaches maturity. We leave this much more complex question to future research.

We maintain simplicity by assuming a common one-period model with no interim cash flows.

⁷We ignore the very small difference in time between the declaration date of a dividend and the payment date. We also assume that the cash is invested in the capital budget immediately at time T . The effects of these factors could be handled with a simple present value adjustment.

⁸In this paper, the notation of a time index in parentheses will identify a value that is established at a prior point. Subscripts are used to denote a random value determined at a particular time. Thus, $B(T)$ is the capital budget to be invested at time T that has been established prior to time T . C_T , however, is the cash actually generated at T . Later we will allow for a stochastic capital budget, which will require the use of the notation B_T , the actual capital expenditure at T .

⁹This variable is the *value* of the stream of future cash flows and is not the current cash flow itself. We do not isolate the current cash flow at t , as this value is not necessary to derive the model and can be assumed to be subsumed within C_t .

¹⁰It is important to also noted that C_t is not the value of the firm, unless it were assumed that all of the firm's assets are, or could easily be made, liquid and effectively available for use to pay dividends. We assume that the firm has fixed assets that it will not, or cannot, liquidate. The assets have some value, which is not necessary to incorporate into this model.

(NPV). Although it may seem inconsistent to have a capital budget with zero-NPV, the assumption is merely used so that positive-NPV projects do not provide a misleading impression of the effects of dividend policy. This approach is, of course, the standard Miller-Modigliani (1961) assumption that enables them to consider the effect of dividend policy on shareholder wealth. Our model is, thus, positioned in the no-tax Modigliani-Miller world, so no results from the model should be interpreted as favoring dividends over capital gains. The model simply provides the value of the dividend component of the stock price. Moreover, we value only the upcoming dividend. Extension to a multi-period world is possible but would require compound options. We defer that subject to future research.

We shall explore how dividends are valued under a variety of assumptions about planned capital expenditures, planned dividends, and the flexibility and willingness to use external financing as a source of funds. We require only that the firm know in advance its dividend policy as defined by whether dividends will be a pure residual, whether a minimum or maximum dividend exists, and by whether its capital budget will be met through external financing if necessary, and whether a maximum dividend is specified.

II. Application of the Model to Various Scenarios

A. *Valuation of Dividends with a Fixed Capital Budget and No External Financing: A Pure Residual Dividend Policy*

As noted above, we start with the assumption that at time t , an all-equity firm determines its capital budget for T as $B(T)$. This amount can be viewed as a maximum capital budget, in that the firm knows its attractive investment opportunities will at least equal this amount. It is possible, however, that the firm will not generate sufficient cash to invest this amount. The firm will follow a residual dividend policy and will not raise additional funds using external financing, an assumption we relax later. If the cash flow at T , C_T , equals or exceeds $B(T)$, the firm will invest $B(T)$ in new projects and pay a dividend of the remaining amount. If the cash flow at T is less than $B(T)$, the firm will invest the full amount of the cash flow and pay no dividend. In this outcome, which can be viewed as the classic case of capital rationing, the firm will be unable to meet its planned capital budget. Table 1 illustrates this scenario. The dividend at time T can be seen as equivalent to a call option expiring at time T in which the underlying is C_T and the exercise price is $B(T)$. The dividend is simply zero or $C_T - B(T)$, which is precisely the payoff structure of a call option.

Although this point is established without requiring any assumptions about the distribution of C_T , we can derive further results by making such assumptions. Let us assume that the cash flow follows the standard lognormal diffusion,¹¹

$$dC_t = C_t \mu_C dt + C_t \sigma_C dW_t,$$

where μ_C is the drift or expected relative change in C_t and σ_C is its volatility. We let the time until the dividend, $T - t$, be represented by τ and let r be the continuously compounded risk-free rate. Then the value at time t of the dividend at time T can be given by the Black-Scholes-Merton (BSM) model. This formula is, of course, well-known and we simply express it in the compact form $D_t(C_t, \tau, B(T))$. Using the Black-Scholes-Merton model, we can now analyze the comparative statics. For example, the following results use the known partial derivatives of the Black-Scholes-Merton model:

$$\begin{aligned} \frac{\partial D(C_t, \tau, B(T))}{\partial C_t} &= N(d_1) > 0 \\ \frac{\partial D(C_t, \tau, B(T))}{\partial B(T)} &= -re^{-r\tau} N(d_2) < 0 \\ \frac{\partial D(C_t, \tau, B(T))}{\partial r} &= \tau B(T) e^{-r\tau} N(d_2) > 0 \\ \frac{\partial D(C_t, \tau, B(T))}{\partial \tau} &= \frac{C_t}{\sqrt{\tau}} \frac{e^{-d_1^2/2}}{\sqrt{2\pi}} > 0 \\ \frac{\partial D(C_t, \tau, B(T))}{\partial \sigma} &= \frac{C_t \sigma_C}{2\sqrt{\tau}} \frac{e^{-d_1^2/2}}{\sqrt{2\pi}} > 0, \end{aligned}$$

where d_1 , d_2 , $N(d_1)$, and $N(d_2)$ are well-known probability terms in the BSM formula.

The first result indicates that the value of the dividend at t is greater, the greater the value of the cash flow at t . This result is logical. A dividend will be paid at T only if the cash flow at T is sufficient to cover the capital budget. The larger the value of the cash flow at t , the more likely are large dividends at T . The second result indicates that the value of the dividend at t is lower the larger is the planned capital budget. Again, this result is logical, because the larger the planned capital budget, the less likely that funds will be available to pay a dividend. If a dividend is paid, it will be smaller the larger is the capital budget. The third result is that the value of the dividend at t is larger, the higher is the risk-free interest rate.¹² As with ordinary call options, this is a

¹¹Again, note that C_t is not the cash flow at t but the value at t of the cash flow at T .

¹²The explanation of this result is not immediately obvious, nor is it obvious for ordinary call options. It is generally argued that a call option is worth more the greater is the interest rate, because a call option is a leveraged investment in the underlying. With a higher interest rate, the call option is more attractive than borrowing to invest in equity. This expiration is somewhat oversimplified but adequate, especially in light of the relative insensitivity of an option value to interest rates.

relatively weak effect so we shall not concern ourselves with it here. The fourth result is that the value of the dividend is greater, the longer the time until the dividend is paid. This result may seem counterintuitive but arises from the fact that as the dividend day draws near, there is less uncertainty about the amount of the dividend. Hence, the value of the potential payout is less. Since the dividend is an option that may or may not be exercised, the less time remaining, the less uncertainty and the lower the value of this option. The fifth result is that the value of the dividend is greater the more volatile is the cash flow. This result should also seem logical, because the upside potential is greater the greater the volatility, while the downside effect is negated by the minimum dividend of zero.

Figure 1 provides an example of the value of the dividend for a firm in which the current value of the cash flow is 100, the volatility is 30 percent, the risk-free rate is 5 percent, the dividend payment date is in one year, and the capital budget ranges from 0 to 200. For example, with a capital budget of 10, the dividend is worth 90.49. With a capital budget of 50, the dividend is worth 52.48. Of course, the capital budget can greatly exceed the current value of the cash flow. There is no requirement that the firm generate sufficient cash flow to meet its capital budget. With a capital budget of 150, the dividend is worth 2.06. With a capital budget of 200, the dividend is worth only 0.23.

This model also permits us to determine a current value for the planned capital budget. We denote this value as $B_t(T)$. If the firm plans to invest $B(T)$ at a future date, the value of that investment at t is easily found as $C_t - D(C_t, \tau, B(T))$. The interpretation of this number is that there is an expected capital budget but that the actual capital budget is uncertain. The expectation is taken under risk neutrality. The comparative statics can be found as follows:

$$\begin{aligned} \frac{\partial B_t(T)}{\partial C_t} &= 1 - \frac{\partial D(C_t, \tau, B(T))}{\partial C_t} = 1 - N(d_1) > 0 \\ \frac{\partial B_t(T)}{\partial B(T)} &= -\frac{\partial D(C_t, \tau, B(T))}{\partial B(T)} = re^{-r\tau} N(d_2) > 0 \\ \frac{\partial B_t(T)}{\partial r} &= -\frac{\partial D(C_t, \tau, B(T))}{\partial r} = -\tau e^{-r\tau} B(T) < 0 \\ \frac{\partial B_t(T)}{\partial \tau} &= -\frac{\partial D(C_t, \tau, B(T))}{\partial \tau} = -\frac{C_t \sigma_v e^{-d_1^2/2}}{2\sqrt{2\pi\tau}} - rB(T)e^{-r\tau} N(d_2) < 0 \\ \frac{\partial B_t(T)}{\partial \sigma} &= -\frac{\partial D(C_t, \tau, B(T))}{\partial \sigma} = -\frac{C_t \sqrt{\tau} e^{-d_1^2/2}}{\sqrt{2\pi}} < 0 \end{aligned}$$

We see that the value of the capital budget increases with the value of the cash flow and the planned capital budget and decreases with the risk-free rate, the time until the funds are invested, and the volatility of the cash flow.

B. Valuation of The Dividends with a Fixed Capital Budget, External Financing and a Target and Possibly Extra and/or Maximum Dividend

Now we consider the possibility that the firm will use external financing to fill any cash shortfall.¹³ This approach is consistent with the well-known pecking order theory of corporate financing. Now, however, we must impose another condition. The firm would use external financing only if it fails to achieve the minimum cash flow necessary to pay its desired dividend and meet its capital budget. Thus, the firm must have a target dividend, which we denote as $D(T)$. If cash flow exceeds the target dividend plus the planned capital budget, the firm will pay an extra dividend of any excess cash. If cash flow falls below the target dividend plus capital budget, the firm will issue new debt or equity to make up the difference to fund its capital budget and pay the dividend.¹⁴ With money being fungible, the funds need not be viewed as being used to meet the dividend or the capital budget specifically. They are simply being used to meet the total dividend and capital budget.

In this scenario, we shall consider the case of a minimum dividend and a second case of a maximum dividend. In the latter case, if there are any excess funds, they are invested in zero net present value projects, such as share repurchase or debt retirement. A minimum dividend will be denoted as $D_{min}(T)$, and the maximum dividend is $D_{max}(T)$.

Table 2 illustrates the case with a minimum dividend (but no maximum) in the form of a sources-and-uses of funds statement. In the right-most column, the firm generates sufficient cash flow to meet its capital budget and pay its target minimum dividend. The excess is paid out as an extra dividend making the total dividend $C_T - B(T)$. In the middle column, the cash flow is insufficient to meet the total of the planned capital budget and minimum dividend, so the firm obtains funds through new financing, meets its capital budget, and pays its minimum dividend.

The total dividend can be expressed as is $Min(D_{min}(T), C_T - B(T))$, which can be written as $D_{min}(T) + Max(0, C_T - D_{min}(T) - B(T))$. Thus, the target dividend can be

¹³It is not necessary to specify if the firm will use debt or equity. Under the perfect market assumptions, there is no economic difference between debt and equity.

¹⁴We could consider the case that if the capital budget is less than the value of the cash flow at T, the firm would use external financing to generate the cash to meet only its capital budget. In that case, however, the dividend would still be zero and the value of the dividend would be the same as in Section IIA.

viewed as a risk-free bond paying $D_{min}(T)$ at maturity T and a call option on the cash flow with an exercise price of $D_{min}(T) + B(T)$. Hence, the value of the dividend at t is

$$D_{min,t} = D_{min}(T)e^{-r\tau} + c(D_{min,t}, \tau, D_{min}(T) + B(T)).$$

The comparative statics are as follows:

$$\begin{aligned} \frac{\partial D_{min,t}}{\partial C_t} &= \frac{\partial D_{min}(C_t, \tau, D_{min}(T) + B(T))}{\partial C_t} = N(d_{1,min}) > 0 \\ \frac{\partial D_{min,t}}{\partial B(T)} &= \frac{\partial D_{min}(C_t, \tau, D_{min}(T) + B(T))}{\partial B(T)} = -re^{-r\tau}N(d_{2,min}) < 0 \\ \frac{\partial D_{min,t}}{\partial D_{min}(T)} &= e^{-r\tau} + \frac{\partial D_{min}(C_t, \tau, D_{min}(T) + B(T))}{\partial D(T)} = e^{-r\tau} - re^{-r\tau}N(d_{2,min}) \\ &= e^{-r\tau}(1 - rN(d_{2,min})) > 0 \\ \frac{\partial D_{min,t}}{\partial r} &= -\tau e^{-r\tau} + \frac{\partial D_{min}(C_t, \tau, D_{min}(T) + B(T))}{\partial r} = -\tau e^{-r\tau} + \tau e^{-r\tau}(D_{min}(T) + B(T)N(d_{2,min})) \\ &= \tau e^{-r\tau}((D_{min}(T) + B(T)N(d_{2,min})) - 1) <=> 0 \\ \frac{\partial D_{min,t}}{\partial \tau} &= -re^{-r\tau} + \frac{\partial D_{min}(C_t, \tau, D_{min}(T) + B(T))}{\partial \tau} = -re^{-r\tau} + \frac{C_t \sigma e^{-d_t^2/2}}{2\sqrt{2\pi\tau}} + re^{-r\tau}((D_{min}(T) + B(T))N(d_{2,min})) \\ &= re^{-r\tau}((D_{min}(T) + B(T))N(d_{2,min}) - 1) <=> 0 \\ \frac{\partial D_{min,t}}{\partial \sigma} &= \frac{\partial D_{min}(C_t, \tau, D_{min}(T) + B(T))}{\partial \sigma} = \frac{e^{-d_t^2/2} C_t \sqrt{\tau}}{\sqrt{2\pi}} > 0 \end{aligned}$$

We see that the current value of the target dividend is greater the higher is the current value of the cash flow, the higher the minimum dividend, and the higher the volatility. These results should seem logical. The value of the dividend is smaller the greater the capital budget. It is technically not possible to sign the effect of the risk-free rate. The longer the time until the dividend, the lower is the value of the claim on the minimum dividend, but the greater is the claim on the extra dividend, which is a call option.

Figure 2 illustrates the value of the dividend for the example used in Section II with a minimum dividend of 10. An interesting result is obtained. We observe that the value of the dividend in the case of a minimum dividend with external financing is nearly the same as the case in Section II, a purely residual dividend policy. For example, in the first case (residual dividend with no external financing), the value of the dividend if the capital budget is 40 is 61.95. In the second case (minimum dividend with external financing), the value of the dividend if the capital budget is 40 is 61.99. The second case is higher, however slightly, because there can be a special dividend. These small differences are even smaller, if the capital budget is smaller. This result is logical.

With a relatively small capital budget, the likelihood of meeting the target dividend and needing external financing is small. The results for these two scenarios are, therefore, likely to be similar. For a large capital budget, however, the results diverge quite significantly. For a capital budget of 100, the dividend is worth 14.23 in the first case and 19.53 in the second.¹⁵ The dividend value is greater in the second case, because there is a target dividend that will be met with certainty. In the first case, there is no target dividend.

As previously shown, the capital budget itself can also be valued but it is risk-free, because the firm will raise sufficient external financing to meet its capital budget of $B(T)$.

In this scenario, the firm could increase its dividend up to the total cash flow minus the capital budget. Suppose the firm feels that it should not increase the dividend beyond an upper limit. We can easily impose this upper limit on our analysis. Let $D_{max}(T)$ be the maximum dividend. Table 3 illustrates the sources and uses analysis.

In the first of the three columns containing the outcomes, we see that the cash flow is below the planned capital budget and minimum dividend. The firm issues new financing to make up the shortage and is able to meet its minimum dividend and planned capital budget. In the middle column, cash flow is sufficient to cover the minimum dividend as well as the capital budget and also pay an extra dividend above the minimum. No new financing is needed. In the third column, cash flow is large enough that funds remain after paying the extra dividend. There is no need for new financing, and there are sufficient funds to repurchase stock or retire debt.

Table 4 shows that the total dividend can be valued as a long call option on the cash flow struck at $B(T) + D_{min}(T)$, a short call option on the cash flow struck at $B(T) + D_{max}(T)$, and a long risk-free bond paying $D_{min}(T)$ at T . Without the bond, this strategy would be referred to as a bull spread. With the bond, the strategy is equivalent to a common strategy known as a collar.¹⁶

¹⁵It is worth noting, however, that we refer to the relative capital budgets as *large* and *small* merely to make a distinction between the two extremes. In fact, when the capital budget is 40 and the current value of the free cash flow is 100, one might easily argue that the capital budget is not at all *small*.

¹⁶Consider an asset priced at S , a call option struck at exercise price X_L with price C_L and another call, struck at exercise price X_H with price C_H . Go long the first call, short the second, and a long a risk-free bond with face value X_L and present value $PV(X_L)$. This strategy is essentially the same as the strategy that replicates the total dividend as described above. Substituting from put-call parity for the call priced at C_L , the value of the strategy can be expressed as $P_L + S - PV(X_L) - C_H + PV(X_L) = S + P_L - C_H$. This combination of the asset, a long put struck at X_L , and a short call struck at X_H is a collar, a strategy designed to protect the position in the asset by buying a put, financing it by selling a call.

Hence, the value of a dividend in this scenario can be seen to equal the value of a bull spread and a risk-free bond. Thus,

$$D_t = D_{min}(C_t, \tau, D_{min}(T) + B(T)) - D_{max}(C_t, \tau, D_{max}(T) + B(T)) + D_{min}(T)e^{-r\tau}$$

The comparative statics are provided below.

$$\begin{aligned} \frac{\partial D_t}{\partial C_t} &= \frac{\partial D(C_t, \tau, D_{min}(T) + B(T))}{\partial C_t} - \frac{\partial D(C_t, \tau, D_{max}(T) + B(T))}{\partial C_t} = N(d_{1,min}) - N(d_{1,max}) > 0 \\ \frac{\partial D_t}{\partial B(T)} &= \frac{\partial D(C_t, \tau, D_{min}(T) + B(T))}{\partial B(T)} - \frac{\partial D(C_t, \tau, D_{max}(T) + B(T))}{\partial B(T)} = -re^{-r\tau}(N(d_{2,min}) - N(d_{2,max})) < 0 \\ \frac{\partial D_t}{\partial D_{min}(T)} &= -re^{-r\tau}N(d_{2,min}) + e^{-r\tau} > 0 \\ \frac{\partial D_t}{\partial D_{max}(T)} &= -\frac{\partial D(C_t, \tau, D_{max}(T) + B(T))}{\partial D(T)} = N(d_{2,max}) > 0 \\ \frac{\partial D_t}{\partial r} &= \tau e^{-r\tau}((D_{min}(T) + B(T))N(d_{2,min}) - (D_{max}(T) + B(T))N(d_{2,max})) - \tau D_{min}(T)e^{-r\tau} <=> 0 \\ \frac{\partial D_t}{\partial \tau} &= \frac{C_t \sigma}{2\sqrt{2\pi\tau}} \left(e^{-(d_1^*)^2/2} - e^{-(d_1^{**})^2/2} \right) + re^{-r\tau}(D_{min}(T) + B(T))N(d_{1,min}) - (D_{max}(T) + B(T))N(d_{1,max}) <=> 0 \\ \frac{\partial D_t}{\partial \sigma} &= \frac{C_t \tau}{\sqrt{2\pi}} \left(e^{-(d_1^*)^2/2} - e^{-(d_2^*)^2/2} \right) <=> 0 \end{aligned}$$

If C_t increases, the value of the dividend increases as it logically should, and as the capital budget increases, the value of the dividend decreases. If the minimum dividend increases, the value of the dividend increases. This result is a combination of the negative effect on the option value of the extra dividend and the positive effect of the minimum dividend, the latter of which is larger. Logically, the value of the dividend increases if the maximum dividend increases. The effects of the risk-free rate, time, and the volatility are ambiguous. These results are not unusual as they are well-known to be associated with option bull spreads.¹⁷ Figure 3 illustrates the effect of the capital budget on the value of the dividend using the same example as in the previous two figures. We see that with a low capital budget the dividend value levels out smoothly to the maximum and minimum dividends.

C. Valuation of Dividends with a Stochastic Capital Budget and No External Financing Under a Residual Dividend Policy

Suppose that we now assume that the firm's capital budget is stochastic. This scenario is potentially more realistic, because capital budgets need not be set in stone.

¹⁷For example, the volatility effect is greatest when an option is at-the-money. Thus, the volatility effect could at times be greater for the short option than for the long option.

Value-creating investment opportunities can arrive randomly. We denote the value of the capital budget at T as B_T , which will evolve randomly over time. Under a residual dividend policy, the firm will either meet its capital budget and pay out any excess in dividends or if cash flow is less than the random capital budget, it will invest only its cash flow, therefore paying no dividend. The results are shown in Table 5.

The value of the dividend at T is equivalent to the payoff of an exchange option in which the underlying is the cash flow and the exercise price is the stochastic capital budget.¹⁸ If the capital budget can be assumed to follow the standard lognormal diffusion,

$$dB_t = B_t\mu_B dt + B_t\sigma_B dW_t,$$

where μ_B and σ_B are the drift and volatility, then the value of the dividend can be obtained using the exchange option pricing model:

$$D_t = D(C_t, \tau, B_t),$$

with the formula as adapted from Margrabe (1978). Note that the cash flow and capital budget can be correlated, which is captured by the variable ρ in the Margrabe formula.¹⁹ This formula requires a composite volatility measure, $\sigma = \sqrt{\sigma_C^2 + \sigma_B^2 - 2\rho\sigma_C\sigma_B}$, which reflects the volatility of the variable $\ln(C_t/B_t)$. The comparative statics are obtained using the results from the exchange option pricing model.

¹⁸Briefly, an exchange option is an option in which the exercise price is stochastic. Consider an asset priced at S_t and an option with an exercise price of X_t , which is random and follows a standard lognormal diffusion process. The payoff at expiration of this option is $Max(0, S_T - X_T)$. That is, the buyer of the option delivers the asset X and receives the asset if $S_T > X_T$. The formula for pricing this option is in Margrabe (1978) and Fischer (1978).

¹⁹Indeed C_t and B_t can be highly correlated as in the oil industry, wherein periods of high oil prices lead to large cash flows and more positive NPV drilling and exploration opportunities.

$$\begin{aligned}
\frac{\partial D_t}{\partial C_t} &= N(d_1) > 0 \\
\frac{\partial D_t}{\partial B_t} &= -N(d_2) < 0 \\
\frac{\partial D_t}{\partial \tau} &= \frac{C_t \sigma e^{-d_t^2/2}}{2\sqrt{2\pi\tau}} > 0 \\
\frac{\partial D_t}{\partial \sigma_C} &= \left(\frac{C_t e^{-d_t^2/2} \sqrt{\tau}}{\sqrt{2\pi}} \right) \left(\frac{\sigma_C - \rho \sigma_B}{\sigma} \right) \langle \Rightarrow \rangle 0 \\
\frac{\partial D_t}{\partial \sigma_B} &= \left(\frac{C_t e^{-d_t^2/2} \sqrt{\tau}}{\sqrt{2\pi}} \right) \left(\frac{\sigma_B - \rho \sigma_C}{\sigma} \right) \langle \Rightarrow \rangle 0 \\
\frac{\partial D_t}{\partial \rho} &= - \left(\frac{C_t e^{-d_t^2/2} \sqrt{\tau}}{\sqrt{2\pi}} \right) \left(\frac{\sigma_C \sigma_B}{\sigma} \right) > 0 \\
\frac{\partial D_t}{\partial r} &= 0
\end{aligned}$$

We see that the dividend is worth more the higher is the cash flow and the longer the time until the dividend is paid and worth less the lower is the current value of the capital budget and the higher is the correlation. The correlation effect is negative because as cash flow increases, the capital budget increases. The positive effect of greater cash on the dividend is offset by the greater need for capital investment. The value of the dividend is unrelated to the risk-free rate. The relationships between the dividend and the volatility of the cash flow, the volatility of the capital budget, and the correlation are indeterminate and depend on the correlation. For example, the value of the dividend is positively related to its volatility and the correlation if $\sigma_C > \rho \sigma_B$. Otherwise, it is positively related to the volatility of the capital budget. Suppose the correlation is perfect positive. Then the value of the dividends is positively related to its volatility if $\sigma_C > \sigma_B$ and positively related to the volatility of the capital budget if $\sigma_C < \sigma_B$. If the correlation is zero, the value of the dividend is positively related to its volatility and to the volatility of the capital budget. If the correlation is perfect negative, the value of the dividends is also positively related to its volatility and the volatility of the capital budget. So, in short, the value of the dividends is directly related to cash flow volatility except in the case where $\rho > \sigma_C/\sigma_B$. If $\sigma_C > \sigma_B$, then this condition cannot hold (ρ cannot exceed 1) and the value of the dividends is directly related to the volatility of the cash flow. If $\sigma_C < \sigma_B$, a relatively high correlation can result in dividends being negatively related to cash flow volatility. The logic behind this

result is that the high correlation lowers the value of the composite volatility measure that reflects the ratio of the cash flow to the capital budget. If σ_c were greater than σ_B , the high correlation would be more than offset by the higher cash flow volatility.

Figure 4 illustrates the value of the dividend under the same assumptions as in the previous examples. In addition, we assume a volatility of the capital budget of 0.2, and a correlation of the cash flow with the capital budget of 0.75. As a point of comparison with the previous examples, in this example we look at the case of a stochastic value of the capital budget of 100. With a risk-free rate of 5% over one year, this would be equivalent to an expected capital budget (under risk neutrality) of 105.13. The dividend value is 14.23. In the initial case, a known capital budget with a residual dividend policy, the value of the dividend with a capital budget of 105.13 would be 11.92. Thus, with a stochastic capital budget, the dividend value is higher than the most comparable case with a known but equivalent (expected) capital budget. Although these cases are not strictly comparable, they are somewhat similar and serve to show the value of randomness when dividends are viewed as contingent claims.

D. Possible Extensions

Another possible scenario is that the dividend could be completely fixed and the capital budget could be allowed to vary. In that manner, the capital budget would be valued similarly to the dividend in the examples here. The model could also be inverted, in the manner of implied volatility, to find the volatility of the cash flow. Moreover, we have looked only at the upcoming dividend and capital budget. Extension to a series of future dividends could be done using compound option pricing theory, but the mathematics would become far more complex. It would also be possible to let excess cash be the residual to be valued, which could be used to retire debt or equity. The current value would indicate the risk-neutral expectation of future debt or equity buybacks. Preferred stock could also be incorporated into the analysis.

III. Conclusions

Corporate dividend decisions and their relationship to capital budgeting can be viewed as options. We see that when companies fix their capital budgets and pay dividends as a pure residual, the dividend is like a call option on the company's cash flows where the exercise price is the capital budget. When companies fix their capital budgets, establish a minimum dividend, and use external financing to meet these objectives, the dividend can be viewed as a combination of a risk-free bond and a call option on the cash flows where the exercise price is the capital budget and the minimum dividend. When companies fix their capital budgets, establish both minimum and

maximum dividends, and use external financing if necessary to meet these objectives, the dividend can be viewed as a combination of a risk-free bond and an option spread, which is long a call struck at the capital budget plus the minimum dividend and short a call struck at the capital budget plus the maximum dividend. Finally, we showed that when the capital budget is stochastic and no external financing is used, the dividend can be viewed as an exchange option, which is an option with a stochastic exercise price, represented by the current value of the capital budget.

Table 1. Fixed Capital Budget, No External Financing, Residual Dividend

Component of Cash Flow	Allocation of Cash Flow at T	
	$C_T < B(T)$	$C_T \geq B(T)$
Capital Investment	C_T	$B(T)$
Dividend	0	$C_T - B(T)$
Total	C_T	C_T

Table 2. Fixed Capital Budget, Minimum Dividend, External Financing

	Allocation of Cash Flow at T	
	$C_T < B(T) + D_{min}(T)$	$C_T \geq B(T) + D_{min}(T)$
Sources of Funds		
Cash Flow	C_T	C_T
New Financing	$B(T) + D_{min}(T) - C_T$	0
Total Sources	$B(T) + D_{min}(T)$	C_T
Uses of Funds		
Capital Investment	$B(T)$	$B(T)$
Minimum Dividend	$D_{min}(T)$	$D_{min}(T)$
Extra Dividend	0	$C_T - B(T) - D_{min}(T)$
Total Uses	$B(T) + D_{min}(T)$	C_T

Table 3. Fixed Capital Budget, Minimum and Maximum Dividends, External Financing

	Allocation of Cash Flow at T		
	$C_T \leq B(T) + D_{min}(T)$	$B(T) + D_{min}(T) < C_T < B(T) + D_{max}(T)$	$C_T \geq B(T) + D_{max}(T)$
Sources of Funds			
Cash Flow	C_T	C_T	C_T
New Financing	$B(T) + D_{min}(T) - C_T$	0	0
Total Sources	$B(T) + D_{min}(T)$	C_T	C_T
Uses of Funds			
Capital Budget	$B(T)$	$B(T)$	$B(T)$
Minimum Dividend	$D_{min}(T)$	$D_{min}(T)$	$D(T)$
Extra Dividend	0	$C_T - B(T) - D_{min}(T)$	$D_{max}(T) - D_{min}(T)$
Debt or Equity Retirement	0	0	$C_T - B(T) - D_{max}(T)$
Total Uses	$B(T) + D_{min}(T)$	C_T	C_T

Table 4. Demonstration that the Value of a Dividend under a Fixed Capital Budget, Minimum and Maximum Dividends, and External Financing is Equivalent to Long and Short Call Options on the Cash Flow and a Risk-Free Bond

	Allocation of Cash Flow at T		
	$C_T \leq B(T) + D_{min}(T)$	$B(T) + D_{min}(T) < C_T < B(T) + D_{max}(T)$	$C_T \geq B(T) + D_{max}(T)$
Dividends			
Minimum Dividend	$D_{min}(T)$	$D_{min}(T)$	$D_{min}(T)$
Extra Dividend	0	$C_T - B(T) - D_{min}(T)$	$D_{max}(T) - D_{min}(T)$
Total Dividend	$D_{min}(T)$	$C_T - B(T)$	$D_{max}(T)$
Call Options on Cash Flow			
Long call struck at $B(T) + D_{min}(T)$	0	$C_T - B(T) - D_{min}(T)$	$C_T - B(T) - D_{min}(T)$
Short call struck at $B(T) + D_{max}(T)$	0	0	$-(C_T - B(T) - D_{max}(T))$
Risk-free bond	$D_{min}(T)$	$D_{min}(T)$	$D_{min}(T)$
Total value of options and bonds	$D_{min}(T)$	$C_T - B(T)$	$D_{max}(T)$

Table 5. Stochastic Capital Budget, No External Financing

Component of Cash Flow	Allocation of Cash Flow at T	
	$C_T \leq B_T$	$C_T > B_T$
Capital Budget	C_T	B_T
Dividend	0	$C_T - B_T$
Total	C_T	C_T

Figure 1. The Value of the Dividend with a Fixed Capital Budget and No External Financing. The current value of the cash flow is 100, the volatility of the cash flow is 30%, the risk-free rate is 5%, and the horizon is one year.

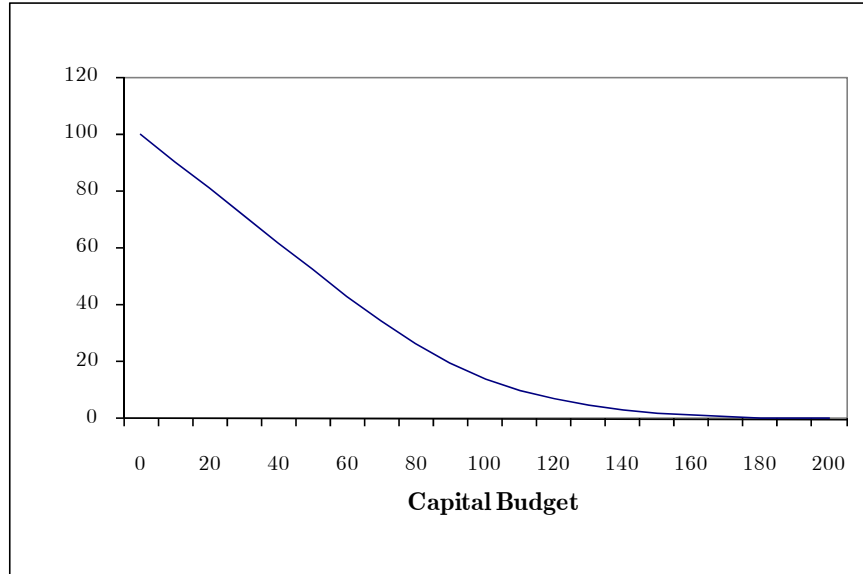


Figure 2. The Value of the Dividend with a Fixed Capital Budget, a Minimum Dividend, and No External Financing. The current value of the cash flow is 100, the volatility of the cash flow is 30%, the risk-free rate is 5%, and the horizon is one year.

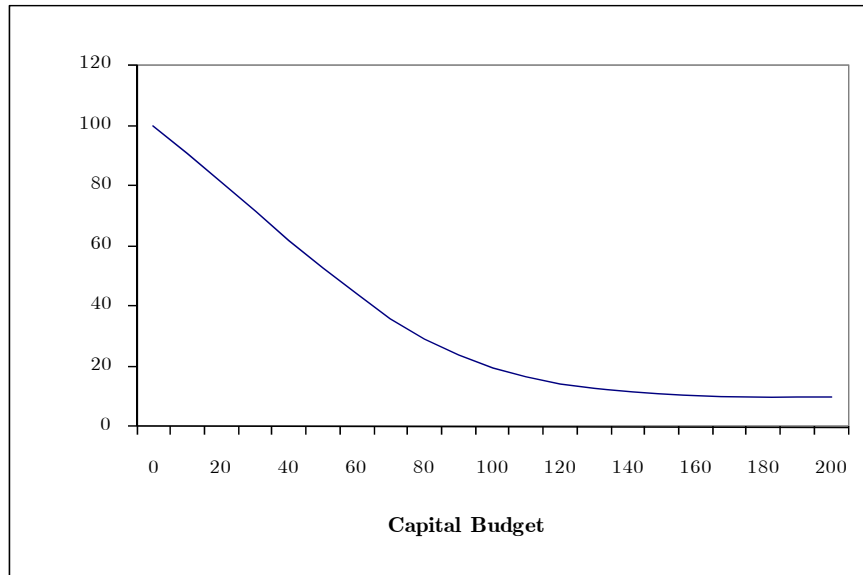


Figure 3. The Value of the Dividend with a Fixed Capital Budget, Minimum and Maximum Dividends, and External Financing. The current value of the cash flow is 100, the volatility of the cash flow is 30%, the risk-free rate is 5%, and the horizon is one year.

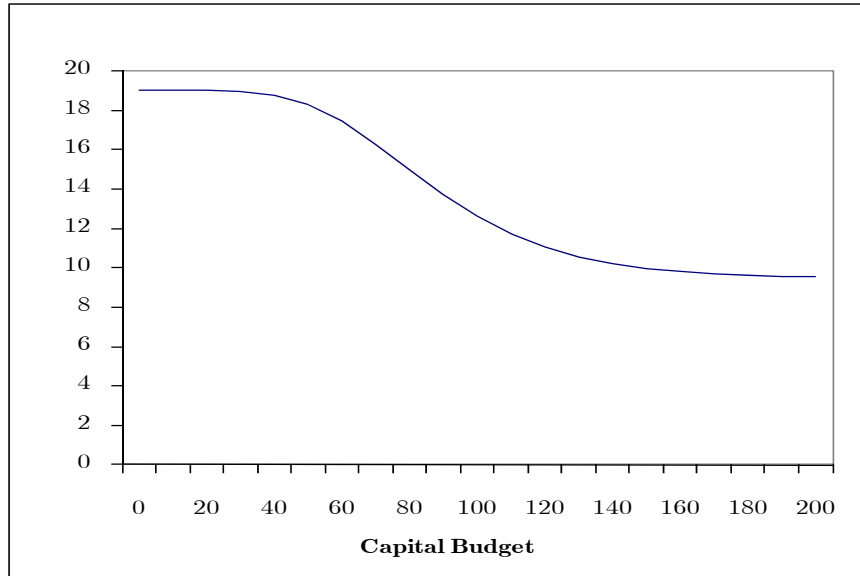
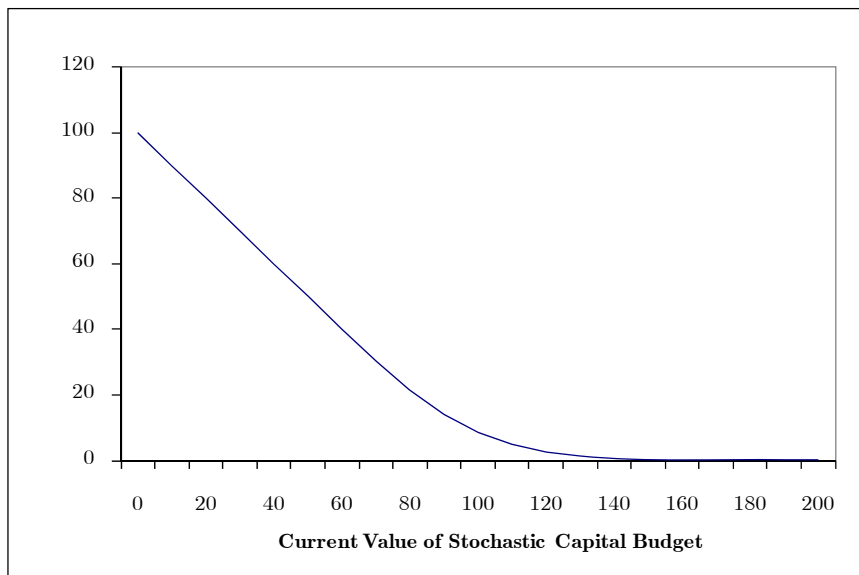


Figure 4. The Value of the Dividend with a Stochastic Capital Budget and No External Financing. The current value of the cash flow is 100, the volatility of the cash flow is 30%, the risk-free rate is 5%, and the horizon is one year.



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