The Determinants of Default Correlations

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Abstract

This paper analyses the ability of some structural models to predict corporate bankruptcy. The study extends the existing empirical work on default risk in three ways. First, it estimates the expected default probabilities and computes default correlations using a copula function for a sample of bankrupt in the US. Second, it extracts common or latent factors that drive companies’ default correlations using a factor-analytical technique. The results indicate that the common factors, which capture the overall state of the economy, explain default correlations. Information-related tests corroborate the results of prediction-orientated tests reported by other studies in the literature; however, only a weak explanatory power is found in the widely used market-to-book assets and book-to-market equity ratio. Idiosyncratic risk observed to change significantly prior to bankruptcy implying that financial markets also react to company specific signals.

JEL: C30; G13; G33.

Keywords: Expected Default Probabilities; Structural Models; Idiosyncratic Risk; Default Correlations
1. Introduction

Corporate defaults exhibit two key characteristics that have profound implications for default risk management. First, default risk is correlated through time. Bankruptcies are normally the end of a process that begins with adverse economic shock and end with financial distress. Although some bankruptcies are unexpected and, therefore, are point events, like Enron and Worldcom, investors become aware of the company’s difficulties some years prior to the bankruptcy event. Second, financial wealth of companies in the same industry, or within the same economic area, is a function of managers’ skills and common factors that introduce correlations.

Companies’ default risk is linked through sector-specific and/or macroeconomic factors. Whilst a great deal of effort has been made by practitioners to measure and explain companies’ default correlations, academics have only recently began to devote attention to this issue. The existing literature on default correlations can be divided into two approaches: the structural approach that models default correlations through companies’ assets values; and the reduced-form approach that models default correlations through default intensities. While financial institutions, namely banks, are aware of these relationships, their ability to model such correlations is still not fully developed. Basle Committee on Banking and Supervision (BCBS 1999, p. 31) states “... the factors affecting the credit worthiness of obligors sometimes behave in a related manner...” which “... requires consideration of the dependencies between the factors determining credit related losses”. Whilst there are many different models and approaches to compute default probabilities, there is no consensus on the importance of different factors that drive default correlations. BCBS (1999) report points out that whilst practitioners have been managing and studying this dependence, there is a lack of theoretical and empirical work on this issue that tests the robustness of the frameworks.

In this paper, we concentrate our empirical investigation on the determinants of default correlation. Our analysis comprises three stages: First, we apply a set of structural models, Merton (M, 1974), Longstaff and Schwartz (LS, 1995) and Ericsson and Reneby (ER, 1998), to compute companies’ expected default probabilities (EDPs). Second, based on cross-sectional tests we analyse the effect of volatility and
idiosyncratic risk on EDPs. Given that unexpected events or fraudulent defaults lead to market-wide jumps in credit spreads, which reduce the ability to diversify this risk, it is important to examine the relationship between company’s idiosyncratic risk and bankruptcy. Third, using a factor-analytical technique, we extract common or latent factors that explain default correlations. This analysis enables us to assess the extent to which default correlation can be ascribed to the latent factors and to the systematic variables from capital and bond markets.

The most popular credit risk frameworks used and sold by financial institutions are the KMV\(^1\) (building on Merton (1974) model) and CreditMetric. According to the Merton model, dependence between companies’ defaults is driven by dependence between assets and threshold values. In the actuarial CreditRisk\(^2\) framework, default correlations are driven by common factors. For each pair of obligors, the asset value is assumed to follow a joint normal distribution. The efficacy of diversification within a portfolio of claims requires accurate estimates of correlations in credit events for all pairs of obligors. For example, Collateralised Bond Obligations (CBO) and valuation of credit derivatives examined by Hull, Predescu and White (2005) require estimates of the joint probability of default over different time periods and for all obligors. Default correlations can lead to a dramatic change in the tails of a portfolio’s probability density function of credit losses (PDCL) and, consequently, in the economic capital required to cover unexpected losses. The common assumption of independence between events produces the right tail of the theoretical PDCL to be thinner than the one observed in practice, which implies that observed unexpected losses are higher than the ones estimated. BCBS (1999) points out that PDCL of portfolios are skewed toward large losses and are more difficult to model. The PDCL that result from the combination of single credit exposures depends on the assumptions made about credit correlations.

The rest of the paper is organised as follows: Section 2, presents a brief digression on dependence measures, with an exposition of copula functions. Section 3 provides a discussion on empirical analyses of structural models and the variables that can account for default correlations and contagion effects. Section 4 contains empirical work.

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1 CreditMetrics was developed by RiskMetrics Group. KMV was developed by KMV Corporation.
2 CreditRisk was developed by Credit-Suisse Financial Products.
Section 5 discusses the implications of the results and section 6 provides some concluding remarks.

2. A Brief Digression on Measures of Dependence

Pearson correlation coefficient, $\rho$, commonly used in finance as a measure of dependence between two variables, assumes that financial variables follow a multivariate normal distribution, which means that it can only be used in the elliptical world (see Embrechtz, McNeil and Straumann (2001) for the limitations of this measure). However, the probability distribution of security returns is not normal, it has fat tails and skewness. This characteristic is crucial for credit risk management, which requires careful consideration of other dimensions of risk. One of these dimensions is the dependence structure between the variables. The copula function allows us to measure this dimension.

In this section, we briefly describe the basic concepts of copula functions.$^3$ A copula function defines the dependence structure between random variables. It links univariate marginals to their multivariate distribution. Consider $p$ uniform random variables, $u_1, u_2, ..., u_p$. The joint distribution function of these variables is defined as

$$ C(u_1, u_2, ..., u_p) = \text{Prob} \{U_1 \leq u_1, U_2 \leq u_2, ..., U_p \leq u_p\} \quad (1) $$

where $C$ is the copula function. Copula functions are used to relate univariate marginal distributions functions, $F_1(x_1), F_2(x_2), ..., F_p(x_p)$, to their joint distribution function

$$ C(F_1(x_1), F_2(x_2), ..., F_p(x_p)) = F(x_1, x_2, ..., x_p) \quad (2) $$

For the random variable, the univariate marginal distribution can be chosen according to its features. The copula function does not constrain the choice of the marginal distribution. Sklar (1959) (cit in Frees and Valdez (1998)) proves that any multivariate distribution function, $F$, can be written in the form of equation (2). He also shows that if

$^3$ A fuller understanding of this subject is set out in Frees and Valdez (1998), Nelsen (1999) and Costinot, Roncalli and Teiletche (2000).
each marginal distribution function is continuous, then there is a unique copula representation.

Copula functions have been used in biological science to analyse the joint mortality pattern of groups of individuals. Li (2000) applied this concept to default correlation between companies. Schonbucher and Schubert (2001) use a different approach, the frailty model, to study default correlations within an intensity model, which is used in biological studies to model heterogeneity via random effects.

The copula summarizes different types of dependencies even when they have been scaled by strictly monotone transformations (invariance property). The properties of bivariate copula functions, $C(u, \upsilon, \rho)$, where $u$ and $\upsilon \in (0, 1)^2$ and $\rho$ is a correlation parameter (it can be Pearson correlation coefficient, Spearman’s Rho, Kendall’s Tau or none of these) are as follows:

(i) since $u$ and $\upsilon$ are positive numbers, $C(0, \upsilon, \rho) = C(u, 0, \rho) = 0$;
(ii) the marginal distribution can be obtained by $C(1, \upsilon, \rho) = \upsilon$ or $C(u, 1, \rho) = u$
(iii) if $u$ and $\upsilon$ are independent variables, $C(u, \upsilon, \rho) = u \upsilon$
(iv) the upper and lower bound for a copula function is $\max(0, u + \upsilon - 1) \leq C(u, \upsilon) \leq \min(u, \upsilon)$

The generalization of these properties to higher dimensions is straightforward.

The joint distribution function is defined by its marginals and the copula. This means that we can examine the copula function to capture the association between random variables. Both Spearman’s Rho, $\rho_S$, and Kendall’s Tau, $\tau$, can be defined in terms of the copula function as follows

$$\rho_S = 12 \int \int [C(u, \upsilon) - u \upsilon] du \upsilon$$

$$\tau = 4 \int \int C(u, \upsilon) dC(u, \upsilon) - 1$$
The non-parametric correlation measures do not depend on the marginal distributions and are not affected by non-linear transformations like the Pearson correlation coefficient.

Mendes and Souza (2004) demonstrate that the copula density function splits the joint distribution function into parameters of the margins, \( \gamma \), and parameters of the dependence structure, \( \delta \). To fit a copula to bivariate data we maximize the log-likelihood function, \( \iota \)

\[
\iota = (u, \upsilon, \gamma_u, \gamma_\upsilon, \delta) = \log [c(F_u(u, \gamma_u), F_\upsilon(\upsilon, \gamma_\upsilon); \delta)] + \log f_u(u; \gamma_u) + \log f_\upsilon(\upsilon; \gamma_\upsilon)
\]  

(5)

where \( c \) is the copula density function and \( f \) is the marginal density function\(^4\).

Durrleman, Nikeghbali and Roncalli (2000) present different methods for choosing the right copula. In this study, we rely on the standard measures, Akaike Information Criterion (AIC) and the Bayesian Information Criterion (BIC).

3. Default Risk and Correlations

Over the past few decades, the dependence within financial markets has been extensively studied both in portfolio diversification and financial integration. Only recently, however, important events like the Asian crisis have spurred academic interest in explaining the causes and consequences of the burst of new economies’ bubble and fraudulent bankruptcies of Enron and WorldCom. More recently, researchers have started to examine the dependence structure between companies’ extreme events.

The structural models attempt to capture of the salient features of the real economy that cause corporate defaults. These models can be divided into two sets: Merton (1974) model, that considers default only at the maturity of the zero coupon bonds; and those models, that allow default to occur at any time within the prediction horizon – the first

\(^4\) See Mendes and Souza (2004) for an example of the fitting process. The authors assume that the margins of IBOVESPA and S&P500 follow a \( t \)-student distribution and fit four copulas: the \( t \)-student, the BB1, the Gumbel and the Gaussian copula.
passage models. We must note that as these models work with risk neutral measures, their expected default probability can differ from the real one and is likely to be higher. This is because the drift of the real process of the asset value is normally higher than the risk neutral process (see Appendix A1). Thus, we expect to observe EDPs resulting from the first passage models to be higher than the one resulting from the first set of models.

The empirical analyses of structural models, including Jones, Mason and Rosenfeld (1984), Huang and Huang (2003) and Eom, Helwege and Huang (2004), report that these models tend to systematically underestimate observed yield spreads and, given the high dispersion of predicted spreads, are inaccurate. In our opinion, this does not affect the accuracy of a structural model in estimating companies’ expected default probabilities. The existing studies on observed yield spreads do not consider all the relevant components that affect yield spreads. As Fisher (1959) argues, an observed bond yield spread provides compensation to investor for credit risk and marketability risk. Several authors (see Delianedis and Geske (2001) and Ericsson and Reneby (2005)) point out that default spread is only a small proportion of the observed yield spread. The studies by Leland (2002), Patel and Vlamis (2004) and Patel and Pereira (2005), report that EDPs from structural models are able to predict bankruptcies, in some cases up to two years before the event. Evidently, the EDPs contain valuable information especially in cases when companies are close to economic/financial distress.

Recently, a number of authors have investigated the extremal dependence of risk factors to model default. Insofar as defaults are infectious, an analysis of default correlations is crucial. Li (2000) is one of the earliest studies to systematically examine default correlations. The author models default correlation between two companies as the correlation between their survival times. He uses the copula concept to define the joint distribution of survival times with given marginal distributions. Li (2000) points out that CreditMetrics uses a bivariate normal copula function with asset correlation as the correlation factor. Laurent and Gregory (2003) extend this work to several obligors. Frey and McNeil (2001) use a copula function and the notion of extremal dependence of risk factors to model default correlations in loan portfolio management. Davis and Lo (2001) study how “infectious defaults” (or contagion effects) can be introduced
within the Binomial Expansion Technique developed by Moody’s. The authors investigate this issue assuming that default correlation among all firms of a CBO is equal and time independent.

Hull and White (2000) develop a method to value vanilla credit default swap with counterparty default risk, which assumes that dependence structure among companies’ defaults follows a multivariate normal distribution. Hull, Predescu and White (2005) extend the previous model to several obligors. They assume that companies’ default threshold has a systematic and an idiosyncratic component. The systematic component is defined as the sensitivity of the threshold to a factor (systematic), common to all firms. Default correlation is defined as the product of each company loading to the systematic variable. Zhou (2001) provides an analytical formula to compute default correlations and joint default probability for the first-passage models. However, the empirical application of this framework to portfolios of loans or bonds becomes cumbersome since it only allows pairwise comparison of obligors.

In another line of investigation, Schonbucher (2003) analyse default correlation spreads through channels other than business ties. Assuming an imperfect market, with asymmetric information, default contagion can arise from information effects, learning effects or updating of beliefs, which means that the default of one company provides information about the default risk of other companies. Collin-Dufresne, Goldstein and Helwege (2003) study default contagion via updating of beliefs, within a reduced-form model. According to the authors, unexpected or fraudulent defaults lead to market-wide jumps in credit spreads, which reduces investors’ ability to diversify this risk. Giesecke (2004) argues that macro-economic variables and operational and/or financial ties can explain default correlations between companies. More specifically, default correlations between companies are due to their dependence on macro-economic variables, which cause cyclical default correlations, and operational and financial relationships with other companies that cause default contagion effects.

Malkiel and Xu (2000) find that investors price idiosyncratic risk because they cannot hold a diversifiable portfolio. Similar evidence is presented by Goyal and Santa-Clara (2002). Arguably, if investors’ ability to diversify risk is limited, idiosyncratic risk is
likely to be an important determinant of default correlation in the period leading up to company’s financial distress.

4. Data and Methodology

The stock price and financial data on a sample of bankrupt companies in the USA used in this study is obtained from the Datastream and Osiris database. The names of bankrupt companies are collected from Moody’s Reports (2003, 2005). A company is classified as bankrupt if it missed or delayed disbursement of interest or principal or if it entered into liquidation, receivership or administration. From sources, we compiled an initial sample of 56 bankrupt companies and 59 bankruptcy events between 1996 and 2004. In order to ensure reliability of the results, we excluded thinly traded companies (when there is more than 10 days without any trade) and companies with less than 5 years of financial data. The remaining sample comprises 34 bankrupt companies, a total of 34 bankruptcy events and 282 yearly observations. For the risk-free rate, we use the yield on 1-year Treasury constant maturity (TCM) securities, from 1990 to 2004, reported by the US government securities dealers to the Federal Reserve Bank of New York.

Our empirical methodology comprises three stages: Stage 1 involves estimation of companies’ EDPs, using three structural models. Prediction-oriented and information-related tests are employed to infer the performance of those models. Stage 2 involves an estimation of companies’ idiosyncratic risk. Stage 3 involves a factor analysis of companies’ default correlation matrix and an analysis of the latent factors.

4.1 Estimation of EDPs
Appendix A1 presents an outline the Merton (M, 1974), Longstaff and Schwartz (LS, 1995) and Ericsson and Reneby (ER, 1998)). Each of these models has a set of parameters that we either estimate or assume to be given. Table 1 describes the parameters and how they are computed in our analysis. Our calibration approach is not very different from the standard one employed in previous studies except that the focus here is solely on the parameters needed to compute the EDPs.
Ideally, to apply these structural models, we should have companies with simple capital structures with only the equity and zero coupon bonds. One practical approach is to assume that company’s debt can be converted to a 1-year zero coupon bond with a face value equal to its debt value. The total market value of the company, and its volatility, can be computed using an iterative procedure based on Ito’s Lemma\(^5\) (a similar procedure used by KMV). For the initial estimate of the company’s volatility, \(\sigma_V\), we compute the standard deviation of daily equity returns, \(\sigma_E\), over the past twelve months. Then, using equation (A4), we compute iteratively the daily market value of the company, \(V_t\), corresponding to the market value of equity, \(E_t\) until the difference in values of \(\sigma_V\) from two consecutive iterations converge to less than 10E-4. Once the convergence has been achieved, the final estimate of \(\sigma_V\) is then used to compute the market value of the company \(V_t\). We consider \(T\) and \(\tau\) to equal one year, assuming that investors’ prediction horizons are one year. The parameter \(\delta\) captures the payments made by the company to its shareholders and bondholders, such as dividends, share repurchases and bond coupons. According to Huang and Huang (2003), 6% can be assumed to be a reasonable estimate for this parameter\(^6\).

Several studies in the literature report that, bondholders’ recovery rate varies according to the seniority of the debt. For example, Altman (1992) (cit in Longstaff and Schwartz (1995)) finds that, during the period 1985-1991, the average recovery rate for a sample of defaulted bond issues was: 0.605 for secured debt, 0.523 for senior debt, 0.307 for senior subordinated debt, 0.28 for cash-pay subordinated debt, 0.195 for non cash-pay subordinated debt. Given this evidence, previous studies (e.g. Longstaff and Schwartz (1995), Leland (2002), Huang and Huang (2003), Eom, Helwege and Huang (2004)) assume an average recovery rate of 51% of debt face value.

\[^5\] We solve Ito’s equations

\[\sigma_V = E_t(V_t, \sigma_V, T-t) / V_t \cdot \sigma_E \cdot N(d_1)\]

and \(E_t(V_t, \sigma_V, T-t) = \hat{E}_t\)

where \(E_t(V_t, \sigma_V, T-t)\) is the theoretical value of company’s assets, \(\sigma_E\) the volatility of equity, \(N(.)\) the standard normal distribution function, and \(\hat{E}_t\) denotes the observed market value of equity.

\[^6\] This value is the weighted average, by the average leverage ratio of all firms for S&P 500, between the observed dividend yield and historical coupon rate (during the period 1973-1998). Huang and Huang (2003) also argue that the use of one payout ratio for firms with different credit ratings is not erroneous given that, probably, firms with lower credit rating may have higher debt’s payouts than the ones with higher credit rating but they also are likely to make less payment to shareholders.
In the one-factor models (M and ER), we use the yield on 1-year TCM rate the risk-free rate. In the two-factor LS model, we assume the interest rate is driven by Vasicek process described in equation (A1). Based on the evidence reported by Eom et al (2004) who apply Vasicek and Nelson-Siegel models to estimate the term structure of the risk free yield curve, we fit Vasicek model to 1-year TCM rates assuming that $\sigma_r = 0.015$ (see Appendix A1 and equation (A2) for details of the estimation procedure). We estimate the parameters $a$ and $\lambda$ using this procedure for each year, from 1990 to 2004, with the daily observations of 1-year TCM. The correlation coefficient is computed with 1-year TCM rates and $V_t$ for each common year.

4.2 Estimation of Idiosyncratic Risk

A widely used procedure for estimating the idiosyncratic risk involves extracting the residuals of an asset pricing model. Obviously, the estimates are sensitive to the chosen asset pricing model and the specified variables. Since the existing literature has tended to employ the three-factor model\(^7\) of Fama and French (1993), we use the following model

$$R_{i,t} - R_{f,t} = \beta_{m,i} (R_{m,t} - R_{f,t}) + \beta_{smb,i} R_{smb,t} + \beta_{hml,i} R_{hml,t} + \epsilon_{i,t} \quad (6)$$

where $R_{i,t}$ is the return on stock $i$ in day $t$, $R_{m,t} - R_{f,t}$ is the market excess return. $R_{smb,t}$ captures the size effect (specified here as the average return on three smallest companies’ portfolios minus the average return on the three largest companies’ portfolio). $R_{hml,t}$ captures the book-to-market equity effect (specified here as the average return on two “value” stocks portfolio minus the average return on two “growth” stocks portfolios). We fit this model using daily observations over the previous year. The standard deviation of $\epsilon_{i,t}$ is then used as the proxy for company’s $i$ idiosyncratic risk.

4.3 Factor analysis of default correlation

Based on equation (5), we fit copula functions to each pair of companies’ EDPs. To fit the copula functions, we define a minimum of 5-year common period for each company. This reduces our sample from 25 to 23 bankruptcy events. The estimated copula functions for each pair of companies’ EDPs (a total of 276 copula functions) are

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\(^7\) We thank Kenneth French for making available this data on his web page: http://mba.tuck.dartmouth.edu/pages/faculty/ken.french.
then transformed into Kendall’s Tau using equation (4). A default correlation matrix is computed for each structural model. In order to extract the common or latent factors that are not directly observable, we use the Kendall’s Tau as the dependent variable, $Facts_t$, in the Factor Analysis (see Appendix A2):

$$Facts_{t,1} = \alpha + \beta_1 X_{1,t} + \ldots + \beta_N X_{N,t}$$

(7)

Where $X_{N,t}$ denote the common or latent factors.

Our next task the identification variables that explain the default correlations. Drawing on the recent, we selected the following set of variables for their theoretical robustness and empirical measurability:

1) *Treasury Interest Rates Level*. Several authors (e.g. Longstaff and Schwartz (1995), Leland and Toft (1996)) argue that an increase spot rate increases the drift of the company’s asset value process and causes EDPs to fall. Since the majority of the models consider the default threshold to be constant or deterministic, an increase in the drift pushes a company’s value away from threshold value and decreases default probability. Since an increase in the level of interest rates decreases EDPs, we should also expect to observe a decrease in default correlations. We use the yield on the 10-year TCM securities $r_{t,10}$, for the interest. In line with Collin-Dufresne, Goldstein and Martin (2001), we consider the square of term, $(r_t^{10})^2$, to capture potential nonlinear effects due to convexity.

2) *Slope of the Yield Curve*. The impact of this variable on default probabilities and default correlations is controversial. In our opinion, since this variable reflects investors’ expectations about the evolution of the economy, an increase in the slope of the yield curve implies strengthening of the economy and, consequently, would lower EDPs and default correlations. We define this variable as the difference between the 10-year and 2-year TCM yields, $r_{t,10} - r_{t,2}$.

3) *Market Volatility*. The market volatility is a critical parameter in structural. The effect of volatility depends on the model specification. In the first-passage model, an increase in volatility increases the probability of default and increases default correlations, because the probability of a company’s value crossing the
threshold at any moment of time also increases. In the European type model, Merton (1974), the effect is not obvious; it can be positive or negative. We measure market volatility, $\sigma_{S&P}$, as the standard deviation of past 12 months of S & P daily returns.

4) **Equity Premium.** Economic intuition suggests that equity premium can be considered a proxy for the overall state of the economy. We expect that an increase in equity premium reflects optimistic economic outlook with lower defaults and default correlations. We measure equity premium, $R_M - r_t^{1m}$, as the spread over the one-month Treasury bill rate of the NYSE Index as well of AMEX and NASDAQ.

5) **Default Return Spread.** This variable captures the systematic risk factor as well as the specific risk when there are unexpected events of bankruptcy or fraud. As explained by Schonbucher (2003), this variable can be interpreted as a learning or information effect variable. An increase in default return spread increases an overall uncertainty in the bond market, which leads to increase in default correlations as investors become more sensitive to bad news. We define default return spread, $DefSpread_t$, as the difference between Moody’s AAA and BAA long term bonds yields.

Table 2 summarizes the expected signs of the relationship between default correlations to the variables outlined above. The first four variables capture cyclical default correlation while the last one captures the systematic component of default contagion effects.

Using the variables explained above, we estimate following regressions equations:

$$Facts_{t, 1} = \alpha + \beta_1 r_t^{10} + \beta_2 (r_t^{10})^2 + \beta_3 (r_t^{10} - r_t^2) + \beta_4 \sigma_{S&P, t} + \beta_5 (R_{M, t} - r_t^{1m}) + \beta_6 DefSpread_t$$

(8)

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8 Collin-Dufresne et al. (2001) define this variable as the difference between BBB Index Yield and 10-year Treasury Yield, which can bias the spread since these two classes of securities have different degrees of liquidity.
5. Empirical Evidence

In this section, we report and discuss the results of three structural models obtained from prediction-oriented and information-related tests. We investigate the importance of idiosyncratic risk in predicting bankruptcy events. We also extract latent factors for companies’ default correlation and identify the determinants of these factors.

Prediction-oriented tests

Prediction-oriented tests provide an in-sample accuracy measure. We classify the results into error type I and II. Since our sample comprises only bankrupt firms, we can just observe Error type I, that is when the model fails to predict bankruptcy. The models correctly predict bankruptcy if in the final available year the EDP is above 20 percent. The results show misclassification of 3 bankruptcy events for M and LS models, which corresponds to 8.3 percent of the bankruptcy events. The performance of the ER model is the best with only 2 misclassified bankruptcy events, which corresponds to 5.5 percent of the sample. From this evidence, we conclude that overall the structural models predict corporate bankruptcy at least one year in advance of the event.

The Summary Statistics of EDPs

Table 3 reports the summary statistics of EDPs for ‘All years’ and for up to n-6 previous years (it is not feasible to present all the results over the period 1990-2004). The first important observation in Table 3, also depicted in Figure 1, is that average EDP of the M model is lower than those of LS and ER models. As mentioned earlier, the former model treats debt as a European option and the latter ones are a kind of Barrier option. Focusing on the behaviour of EDPs, we observe a gradual increase in EDPs up to two years ahead of bankruptcy and then a steep rise a year before the event. The second important observation is that the standard deviations of EDPs of the M model (approximately 25 percent) are comparatively lower than those of LS and ER models (approximately 60 percent and 40 percent, respectively). This suggests that M model is more accurate in predicting bankruptcy than the other two models (see Figures 2 to 4). For the first passage LS model, we observe a distinct clustering, however, this model appears to be the list accurate with more extreme values (see Figures 3 and 4).

Moody’s Report (2005) presents default rates term structure for several period of time. Default probability of a Caa-C firm, during the period 1920-2004, at 1-year horizon, was around 15 percent. For the period 1983-2004, at the same horizon, was around 22 percent. Standard & Poor’s transition probability from CCC to default is around 19.8 percent (see Crouhy et al. (2000)).
Overall, the results suggest that the first passage LS model does not add value over and above the M model. Surprisingly, the two-factor LS model has the worst performance, suggesting that the effort to capture the real world in this model, as far as expected default probabilities are concerned, is not justified.

*Cross-section Analysis of EDPs*

Tables 4 to 6 present the results of the multivariate linear regression model. The EDPs of the structural models are expected to be closely related to two parameters: the volatility and the debt ratio. In table 4, all the coefficients of these two variables are statistically significant and have the expected sign. The volatility and debt ratio explain a high percentage of the variability of EDPs (only in the LS model, this percentage is below 50 percent). The EDPs are more sensitive to the debt ratio than to the volatility. The first passage models are more sensitive to debt ratio than the M model. All regressions are statistically significant at 1 percent level.

Table 5 reports the results for the measure of idiosyncratic risk. The explanatory power of the regressions increases substantially (approximately 70 percent). The debt ratio and idiosyncratic risk explain approximately 77 percent of the variability of EDP of the M model. These results confirm the significance of the idiosyncratic risk in explaining bankruptcy.

Table 6 presents the results of the tests incorporating the all three variables, debt ratio, idiosyncratic risk and volatility. Compared to the results in Table 5, the explanatory power of the regressions has not improved. It is clear, however, that idiosyncratic risk is the most important variable. The coefficient of idiosyncratic risk is statistically significant. It is surprising to observe the coefficient volatility is smaller in the LS and ER model its coefficient is not statistically different from zero.

*Information-related tests*

This analysis complements the prediction-oriented tests. There are, however, several limitations of this method: First, it assumes a dichotomous decision. Second, both error types are equally important. For a credit risk manager, it is more serious to have a
bankrupt firm classified as non-bankrupt than a non-bankrupt firm classified as bankrupt. Third, the classification of firms as bankrupt or as non-bankrupt is somewhat subjective because it implies the definition of a cut-off value. Forth, it is not clear which model explains better the variability of companies’ default risk. Moreover, this procedure has the limitation of considering bankruptcy as an event and not as a process\textsuperscript{10}.

Based on Shumway (2001), we assume that the relationship between companies’ EDPs and independent variable(s) is represented by a logistic curve that asymptotically approaches one (zero) as covariates tend to positive (negative) infinity. This relationship is written as follows\textsuperscript{11}

\[ P_{t-1}(Y_{it} = 1) = \frac{1}{1 + \exp(- (\alpha + \beta X_{i,t-1}))} \]

where \( X_{i,t-1} \) is the vector of time varying covariates, known at the end of previous year, \( \alpha \) denotes the constant, \( Y_{it} \) is the dependent variable, EDPs, and which equals one when a company goes bankrupt and zero otherwise. Each year that a company is alive corresponds to an observation in the estimation equation.

Table 7 reports the results of logistic regressions. Column 1 to 8 displays univariate regressions with EDPs of the structural models\textsuperscript{12} and with idiosyncratic risk, debt ratio, market-to-book assets ratio (MBA), book–to–market equity ratio (BME) and volatility. According to Vassalou and Xing (2004), default risk is explained by the BE ratio, while MBA ratio is introduced as a proxy for companies’ growth opportunities. We use Nagelkerke R\textsuperscript{2} as an indicator of the explanatory power.

All Models are statistically significant and the coefficients have the expected signs. The M model has the highest explanatory power (around 40 percent), which is in contrast to

\textsuperscript{10} One way to solve this problem is to use lag values on the logistic regression. We did not use this procedure because it would entail loss of observations and because fixing a number of lagged values introduces bias.

\textsuperscript{11} This non-linear relationship can be rewritten as a linear one

\[ \ln[P_{t-1}/(1 - P_{t-1})] = \alpha + \beta X_{i,t-1} \]

where the dependent variable represents the log of the odds.

\textsuperscript{12} Based on Patel and Pereira (2005), we also perform logistic regressions with model-scores. We do not show these results because they are very similar to the ones reported.
the results of the prediction-oriented tests reported earlier. The ER model also shows good performance and is very close to the M model. The LS model has comparatively lower performance. Overall, the performance of these models is better than that reported by Campbell Hilscher and Szilagyi (2004) at the one-month horizon. Considering that the financial and accounting data for some failed companies have lags up to two years, these results are highly encouraging.

It is worth pointing out the performance of coefficients idiosyncratic risk and debt ratio. Both these variables have the expected sign and are significant at 1 percent confidence level. The default risk, however, appears not to be sensitive to volatility, and the explanatory power of this coefficient is almost zero. These results show idiosyncratic risk is an important variable. Comparing the explanatory power of idiosyncratic risk (column 4) and volatility (column 8), it is evident that the former variable predicts bankruptcy events better than the latter variable. This suggests that investors are aware of the specific circumstances responsible for company’s deterioration and anticipate bankruptcy. The coefficient of Debt ratio, is significant as expected. It is worth noting the explanatory power of the ER and LS model, which given the complexity of these models is somewhat intriguing. In contrast to the results reported by Vassalou and Xing (2004), the explanatory power of the BME ratio is almost zero, and this variable is not significant. The MBA ratio has the expected sign and is significant at 5 percent confidence level, but its explanatory power is very low (see column 6 and 7).

In column 9 and 10, we report the coefficients of idiosyncratic risk and debt ratio for a stepwise logistic regression of the M and ER model. The idiosyncratic risk always dominates other variables and in M and ER models it is statistically more significant than debt ratio. In both regressions, the explanatory power increases substantially.

Finally, according to –2LogL statistic, that has a \( \chi^2 \) distribution with \( n-q \) degrees of freedom, where \( q \) is the number of parameters in the model, we cannot reject the null hypothesis of logistic regressions, implying that model fits the data.
**Factor Analysis of Correlation Matrix**

Next we present results of the joint variability of companies default risk, that is, of the companies default correlation matrix, based on Factor-analytical tests (see Appendix A2 for details). We compute a correlation matrix per model and fit copula functions\(^\text{13}\) by maximizing the log-likelihood function as explained in equation (4) to each pair of EDPs. Given the restrictions outlined in Section 4 above, the sample comprises 23 companies, which are 25 bankruptcy events and 276 copula functions. The results show that all fitted copula functions belong to the normal family. Next, we construct companies’ correlation matrix with Kendall’s Tau. This correlation matrix is used in the Factor Analysis to estimate the determinants of default correlations.

We employ the principal components method to extract the factors from the correlation matrix. We retain the factors that have an eigenvalue greater than one. Table 8 reports the results of factor analysis for each model. We extract 5 factors for the M and 6 factors the LS and ER model. The RMSR and the non-redundant residuals of the residual matrix are small in all models, implying a good factor solution. The 5 factors of the M model and the 6 factors of LS and ER models, referred to as common or latent factors, explain a high percentage of the observed variance (79.5 percent and 86.5 percent and 82.8 percent, respectively). This is an encouraging result for our search for the determinants of default correlation. In contrast to Zhou (2001), the results suggest that only a small percentage (21.5, 13.5 and 17.2 percent for M, LS and ER models, respectively) of observed variance or default correlation is explained by non-systematic factors. An orthogonal rotation (Varimax rotation) is performed to achieve a simpler factor structure. We use the rotated component matrix to estimate time series values for each model’s factors.

Since it is more difficult to interpret and analyse the determinants of default correlations with so many factors, we use principal component analysis to reduce the factors. We use the rule of eigenvalue greater than one to retain the new factors. The initial 5 common factors of the M model and 6 common factors of LS and ER models are reduced to 2 common factors for each model. The 2 common factors explain around 90 percent of the total variability of its initial common factors (see Table 9).

---
\(^{13}\) Several copula families were also fitted including the normal and extreme values families.
Table 10 presents the determinants of default correlation factors. We estimate equation (6) using a stepwise procedure\textsuperscript{14} because of a multicollinearity problem. As expected, given the assumptions of the stepwise procedure, all the variables and all regressions are statistically significant, at 5 percent confidence level. Overall, the regressors’ explanatory power is very high, around 55 percent, with a maximum of 71 percent. Further, the signs of the estimated coefficients are generally as expected (see Table 2). Market volatility and equity premium explain 56 percent of the variability of the common factor 1 in the M model. This factor can be interpreted as the capital market effect. Equity premium has the expected sign and market volatility has a negative effect on default correlation. Surprisingly, volatility does not explain the variability of default correlation in the first passage models. The slope of the yield curve explains the variability of the common factor 2 in the M model, but the sign of the estimated coefficient sign is not as expected. One possible explanation for this is that an increase at the slope of the yield curve makes it more difficult for distress firms to renegotiate the debt and possibly increases the default risk and default correlations. We observe some similarity in the variables that explain the common factors in the LS and ER models. Both common factors 1 are explained by the slope of the yield curve, although this variable does not have consistent sign in the regressions. Only the common factor 1 in ER model has the expected sign. Both common factors 2 are explained by the Treasury interest rates and market equity premium that allows us to interpret them as a return driven factor. Consistent with Longstaff and Schwartz (1995) and Collin-Dufresne et al. (2001), we find that the effect of increase in the risk free rate is to lower EDPs and default correlations. The estimated coefficients of equity premium are of the same magnitude in all models. The default return spread, proxy for default contagion, is not significant in any of the regressions, suggesting that either it does not explain default correlations, as argued by Schonbucher (2003) and Collin-Dufresne et al. (2003), or that this variable is not the good proxy. We should point out that, given the nature of this effect, it is probably better to capture this effect by a non-systematic variable or a variable that considers companies’ business and financial ties. Convexity is not significant in any of the regressions, which is consistent with the findings of

\textsuperscript{14} Several studies (e.g. Collin-Dufresne et al. (2001) argue that default probabilities can be explained by nonlinear, cross term and lagged values of regressors (such as squared and cubic slope of the yield curve or $(r_t^{10} - r_t^2)S&P$). Nevertheless, none of these terms seems to explain default correlations and that is why we restrict this analysis to the variables in equation (6).
Collin-Dufresne et al. (2001). Ljung-Box test indicates that standardized residuals from the regressions are not autocorrelated, the average serial correlation of standardized residuals is 0.02, and the average Durbin-Watson statistic is 1.81.

In summary, default correlations are driven essentially by common factors that explain on average around 83 percent of total variance. Only a small percentage of default correlation is due to non-systematic factors. These results are consistent with economic intuition and empirical evidence (see Vassalou and Xing (2004)) according to which, during recession periods, default risk increases and a cluster of bankruptcy events is observed. The factors driving default correlations are the capital market equity premium and Treasury interest rates, which reflects the overall state of the economy. This is consistent with the theoretical intuition of Hull et al. (2005) when they argue that the systematic variable that drives default correlations is capital market wiener process. Second, the slope of the yield curve reflects investors’ expectations about the evolution of the economy. So far as default correlations are driven basically by systematic factors, portfolio diversification should reduce default risk.

6. Conclusion

In this study we analyse the determinants of default correlations for a sample of US bankrupt companies. We apply a set of structural models (Merton (1974), Longstaff and Schwartz (1995) and Ericsson and Reneby (1998)) to estimate companies’ EDPs. Given that we observe a sharp increase in EDPs up to two years in advance of default event, these models provide timely and accurate estimates of companies default risk. Another novel finding is the importance of idiosyncratic risk (and not of total volatility) in predicting default events. This suggests that company specific signals provide useful information to investors about the deterioration in company’s economic and financial conditions prior to bankruptcy.

We compute the companies’ default correlation matrix using a copula function and employ Factor Analysis technique to extract factors that explain default correlations. The results of prediction-oriented tests suggest that ER model is the best model as it misclassifies only 5.5 percent of bankruptcy events. The results of information-related
tests suggest that M and LS model have a similar performance. Variables such as market-to-book asset ratio and book-to-market equity ratio, which other studies have found to be significant, have poor explanatory power in our regression analysis. We observe that common factors explain around 83 percent of the variability of default correlations. This evidence supports the belief that common factors are explained by the overall state of the economy and by the expectations of its evolution.
References

Huang, J. and M. Huang, 2003, How much of the Corporate Treasury Yield Spread is Due to Credit Risk, Working Paper, Penn State University.
Appendix A1: Structural Models

In this section, we present a brief summary of the models by Merton (1974), Longstaff and Schwartz (1995) and Ericsson and Reneby (1998). Since our main concern here is with the empirical performance of these models, we do not discuss in detail the theoretical properties of the models. Throughout this section, we assume that uncertainty in the economy is modelled by a filtered probability space \((\Omega, \mathcal{G}, P)\), where \(\Omega\) represents the set of possible states of nature, \(\mathcal{G}_t\) is the information available to investors over time \(t\) and \(P\) is the probability measure. All models assume a perfect and arbitrage-free capital market, where risky and default-free bonds and companies’ equity are traded. The risk-free numeraire (or money market account) value, at time \(t\), \(A_t\), follows the process

\[ A_t = \exp\left( \int_0^t \mathcal{r}_s \, ds \right) \]

where \(r\) denotes the short-term risk-free interest rate, which can be deterministic or modelled by a stochastic process. When modelled as a stochastic process, the dynamics of \(r\) is driven by a Vasicek-model

\[ dr_t = a(\lambda - r_t)dt + \sigma_r dW^r \]  

(A1)

where \(a\) is the short term interest rate mean reversion speed, \(\lambda\) and \(\sigma_r\) are its mean reversion level and standard deviation, respectively. The variable \(dW^r\) is a Wiener process. In this economy, the investors are assumed to be risk-neutral, which means that the probability measure, \(P\), is a martingale with respect to \(A_t\). The value of a riskless discount bond that matures at \(T\) is (Vasicek, 1977):

\[ D(r, T) = \exp(A(T) - B(T)r) \]  

(A2)

\[ A(T) = \left( \frac{\sigma_r^2}{2a^2} - \lambda \right) T + \left( \frac{\sigma_r^2}{a^3} - \frac{a\lambda}{a^2} \right) \left( \exp(-aT) - 1 \right) - \left( \frac{\sigma_r^2}{4a^2} \right) \left( \exp(-2aT) - 1 \right) \]

\[ B(T) = \frac{\left[ 1 - \exp(-aT) \right]}{a} \]
Under the risk neutral probability space, the value of the company’s assets, $V$, follows a geometric brownian motion ($\mathcal{G}_t$ – adapted diffusion process) given by

\[
dV_t = (r_t - \delta)V_t dt + \sigma_V V_t dW^V
\]

(A3)

where $\delta$ denotes company’s assets payout ratio and $\sigma_V$ company’s assets volatility. The variable $dW^V$ is a Wiener process under the risk-neutral probability measure. $\rho$ is the instantaneous correlation coefficient between $dW^f$ and $dW^v$.

The dynamics of company’s assets value, under the real probability space, is given by

\[
dV_t = (\mu - \delta)V_t dt + \sigma_V V_t dW^{pv}
\]

where $\mu$ denotes company’s assets expected total return and $dW^{pv}$ is a Wiener process under the real probability measure. For the dynamic process described by equation (A3), and the given assumptions, the standard hedging framework leads to the following partial differential equation

\[
\frac{1}{2} \sigma_V^2 V^2 \frac{\partial^2 F}{\partial V^2} + (r - \delta)V \frac{\partial F}{\partial V} - rF + \frac{\partial F}{\partial t} + P = 0
\]

where $F$ is the price of any derivative security, whose value is a function of the value of the firm, $V$, and time, and $P$ represents the payments received by this security. The two-factor models by Longstaff and Schwartz (1995) assume that $F$ is a function of the value of the firm, $V$, time and interest rates.

The standard hedging framework leads to the following partial differential equation

\[
\frac{1}{2} \sigma_V^2 V^2 \frac{\partial^2 F}{\partial V^2} + \rho \sigma_V \sigma_r V \frac{\partial F}{\partial V \partial r} + \frac{1}{2} \sigma_r^2 \frac{\partial^2 F}{\partial r^2} + (r - \delta)V \frac{\partial F}{\partial V} + (a\lambda - ar) \frac{\partial F}{\partial t} + \frac{\partial F}{\partial t} - rF + P = 0
\]
Given our focus on the empirical performance of structural models in predicting corporate failure, we only outline the equations relevant for the expected default probability in each model. We refer the reader to the original papers for the full description the models.

**Merton (M) Model:** Merton (1974) model is an extension of the Black and Scholes (1973) option pricing model to value corporate securities. The company’s assets value, which corresponds to the sum of the equity and debt values, is driven by the process described by equation (A3) and is assumed to be independent of company’s capital structure. Under these assumptions, equity value, $E_t$, is defined by a call option on the assets of the firm, with maturity $T$ and exercise price $F$:

$$E_t = V_t N(d_1) - e^{-r(T-t)} F N(d_2)$$  \hspace{1cm} (A4)

where

$$d_1 = \frac{\ln \left( \frac{V_t}{F} \right) + \left( r + 0.5\sigma^2 \right)(T-t)}{\sigma \sqrt{T-t}} , \quad d_2 = d_1 - \sigma \sqrt{T-t}$$

and $N(.)$ represents the standard normal distribution function. Debt’s value, at time $t$, is equal to:

$$D_t = V_t - E_t$$

If at maturity, company’s assets value, $V_T$, is higher than the face value of its debt, $F$, the firm does not default, bondholders receive $F$ and shareholders $V_T - F$. However, if $V_T < F$, the firm defaults and there is a transfer of company’s ownership from shareholders to bondholders. Firm only defaults at time $T$, and $N(-d_2)$ represents the risk-neutral probability of default.

**Longstaff and Schwartz (LS) Model:** Longstaff and Schwartz (1995) develop a two factor model to value risky debt, extending the one-factor model of Black and Cox (1976) in two ways: (i) incorporating both default risk and interest rate risk; (ii) allowing for deviations from strict absolute priority. An important feature of this model is that firms with similar default risk can have different credit spreads if their assets
have different correlations with changes in interest rates. Their assumptions are not very different from the ones used by Black-Scholes, Merton (1974) and Black and Cox (1976), except for the fact that short term risk free interest rate follows the dynamics described by equation (A1) (and the riskless discount bond can be priced using equation (A2)) and that there are bankruptcy costs, $\alpha$. The default boundary, $K$, is constant and exogenously specified, which is consistent with the assumption of a stationary capital structure. Setting $X$ equal to the ratio $V/K$, the price of a risky discount bond that matures at $T$ is

$$D(X, r, T) = D(r, T) - \alpha D(r, T)Q(X, r, T)$$

(A5)

where

$$Q(X, r, T, n) = \sum_{i=1}^{n} q_i$$

(A6)

and

$$a_i = \frac{-\ln X - M(iT/n, T)}{\sqrt{S(iT/n)}}$$

$$b_{ij} = \frac{M(jT/n, T) - M(iT/n, T)}{\sqrt{S(iT/n)} - S(jT/n)}$$

The term $Q(X, r, T)$ is the limit of $Q(X, r, T, n)$ when $n \to \infty$ (the authors argue that the convergence between these terms is rapid and that when $n = 200$, the differences between the results of the terms are virtually indistinguishable).

The first term in equation (A5) represents the value of a riskless bond. The second term represents a discount factor for the default of the bond. The factor can be decomposed into two components: $\alpha D(r, T)$ is the present value of the writedown on the bond if default occurs; $Q(X, r, T)$ is the probability, under the risk neutral measure, that a default occurs (this probability can differ from the real one).
**Ericsson and Reneby (ER) Model:** Ericsson and Reneby (1998) demonstrate that corporate securities can be valued as a portfolio of three basic claims: a down-and-out option that expires worthless if the underlying variable reaches a pre-specified lower boundary, prior to the expiration date; a down-and-out binary option that yields a unit payoff at the expiration date if the underlying asset exceeds the exercise price; and unit down-and-in option that pays off one unit the first time the underlying variable reaches a lower boundary. This formulation allows to value finite maturity coupon debt with bankruptcy costs, corporate taxes and deviations from the absolute priority rule. The default is triggered if company’s value falls below a constant $K$ (the reorganization barrier), at any time prior to maturity of the firm, or if, at debt’s maturity, company’s value is less than some constant $F$, which normally is debt’s face value. The time of default is denoted $\tau$. The price of a unit down-and-in option, that matures at $T$ and pays one monetary unit if bankruptcy happens before $T$ and zero otherwise, is

$$ G^K \{V_t, t | \tau \leq T \} = G^K \{V_t | \tau \leq \infty\}(1 - Q^G \{\tau > T, V_t > K\}) \quad \text{(A7)} $$

where

$$ G^K \{V_t | \tau \leq \infty\} = (V_t / K)^\theta $$

$$ Q^G \{\tau > T, V_t > K\} = N \left\{ d_T^G \left( \frac{V}{K} \right) \right\} - N \left\{ d_T^G \left( \frac{K^2}{V_K} \right) \right\} $$

$$ d_T^G(x) = \frac{\ln x}{\sigma \sqrt{T}} + \mu^G \sqrt{T} $$

$$ \mu^G = \frac{r - \delta - 0.5\sigma^2}{\sigma} \quad \mu^G = \mu^G - \theta \sigma \quad \theta = \frac{\sqrt{(\mu^G)^2 + 2r + \mu^G}}{\sigma} $$

Equation (A10) represents the expected default probability.
Appendix A2: Factor Analysis

A full understanding of Factor Analysis can be done in Sharma (1996). Factor Analysis use the correlation matrix to: identify the smallest number of common factors (via factor rotation) that best explain the correlation among the variables; and provide an interpretation for these common factors.

This technique assumes that the total variance of a variable can be divided into the variance explained by the common factor and the one explained by a specific factor. A factor model that contains $m$ factors can be represented as

$$x_1 = \lambda_{11}\xi_1 + \lambda_{12}\xi_2 + \ldots + \lambda_{1m}\xi_m + \varepsilon_1$$
$$x_2 = \lambda_{21}\xi_1 + \lambda_{22}\xi_2 + \ldots + \lambda_{2m}\xi_m + \varepsilon_2$$
$$\vdots \quad \vdots \quad \vdots \quad \vdots \quad \vdots \quad \vdots \quad \vdots \quad \vdots \quad \vdots$$
$$x_p = \lambda_{p1}\xi_1 + \lambda_{p2}\xi_2 + \ldots + \lambda_{pm}\xi_m + \varepsilon_p$$

where $x_1, x_2, \ldots, x_p$ are variables of the $m$ factors, $\lambda_{pm}$ is the pattern loading of the $p^{th}$ variable on the $m^{th}$ factor and $\varepsilon_p$ is the specific factor for the $p^{th}$ variable. The previous construct can be represented as

$$x = \Lambda\xi + \varepsilon$$ \hspace{1cm} (A8)

$x$ is a $p \times 1$ vector of variables, $\Lambda$ is a $p \times m$ matrix of factor pattern loadings, $\xi$ is a $m \times 1$ vector of latent factors and $\varepsilon$ is a $p \times 1$ vector of specific factors. Equation (A8) is the factor analysis equation. The assumptions are: the common factors are not correlated with the specific factors and the means and variances of variables and factors are zero and one, respectively. Variables’ correlation matrix, $R$, is

$$R = \Lambda\Phi\Lambda' + \Psi$$ \hspace{1cm} (A9)

$\Lambda$ is the pattern loading matrix, $\Phi$ is factors’ correlation matrix and $\Psi$ is a diagonal matrix of the specific variances. $R - \Psi$ gives us the variance explained by the common factors. The off-diagonals of $R$ are the correlation among variables. Factor analysis estimate parameter matrices given the correlation matrix. The correlation between the variables and the factors is given by

$$\Lambda = \Lambda\Phi$$

If the $m$ factors are (not) correlated, the factor model is referred to as an oblique (orthogonal) model. In an orthogonal model, it is assumed that $\Phi = I$. Orthogonal
rotation technique implies the identification of a matrix, $C$, such that the new loading matrix is given by $\Lambda^* = \Lambda C$ and $R = \Lambda^* \Lambda^{**}$.

Varimax rotation technique estimate matrix $C$ such that each factor will by a set of different variables. This is achieved by maximizing the variance of the squared loading pattern across variables, subject to the constraint that the communality of each variable is unchanged. $C$ is obtained maximizing the following equation, subject to the constraint that the common variance of each variable remains the same.

$$pV = \sum_{j=1}^{m} \sum_{i=1}^{p} \lambda_{ij}^4 - \frac{\sum_{j=1}^{m} \left( \sum_{i=1}^{p} \lambda_{ij}^2 \right)^2}{p}$$

where $V$ is the variance explained by the common factors.
Table 1: Estimation of Model Parameters

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Model</th>
<th>Estimated as</th>
</tr>
</thead>
<tbody>
<tr>
<td>Firms’ Specific Parameters</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$V_t$ Company’s Value</td>
<td>All</td>
<td>Ito’s lemma</td>
</tr>
<tr>
<td>$\sigma_V$ Company’s Volatility</td>
<td>All</td>
<td>Ito’s lemma</td>
</tr>
<tr>
<td>$F$ Debt’s Face Value</td>
<td>M book value of total liabilities</td>
<td></td>
</tr>
<tr>
<td>$T$ Years to Maturity</td>
<td>All</td>
<td>assumed 1 year</td>
</tr>
<tr>
<td>$\delta$ Payout Ratio</td>
<td>ER</td>
<td>assumed at 6 percent</td>
</tr>
<tr>
<td>$\tau$ Prediction Horizon</td>
<td>ER</td>
<td>assumed 1 year</td>
</tr>
<tr>
<td>$\alpha$ Bankruptcy Costs</td>
<td>LS</td>
<td>Assumed at 49 percent</td>
</tr>
<tr>
<td>$K$ Threshold Value / Distress Barrier</td>
<td>LS; ER</td>
<td>debt’s face value</td>
</tr>
<tr>
<td>Interest Rate Parameters</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$r$ Interest Rate</td>
<td>All</td>
<td>1-year TCM</td>
</tr>
<tr>
<td>$a$ Mean Reversion Speed</td>
<td>LS</td>
<td>Vasicek risk free yield curve</td>
</tr>
<tr>
<td>$\lambda$ Mean Reversion Level</td>
<td>LS</td>
<td>Vasicek risk free yield curve</td>
</tr>
<tr>
<td>$\sigma_r$ Short Rate Standard Deviation</td>
<td>LS</td>
<td>Assumed at 1.5 percent</td>
</tr>
<tr>
<td>$\rho$ Correlation Coefficient between $r$ and $V_t$</td>
<td>LS</td>
<td>computed</td>
</tr>
</tbody>
</table>

Table 2: Expected Signs of the effect of Explanatory Variables on Default

Correlation

$$Facts_t = \alpha + \beta_1 r_t^{10} + \beta_2 (r_t^{10})^2 + \beta_3 (r_t^{10} - r_t^2) + \beta_4 \sigma_{S&P,t} + \beta_5 (R_{M,t} - r_t^{1m}) + \beta_6 DefSpread_t$$

<table>
<thead>
<tr>
<th>Variable</th>
<th>Predicted Sign</th>
</tr>
</thead>
<tbody>
<tr>
<td>$r_t^{10}$ Yield on 10-year TCM</td>
<td>-</td>
</tr>
<tr>
<td>$r_t^{10} - r_t^2$ 10-year TCM minus 2-year TCM yields</td>
<td>-</td>
</tr>
<tr>
<td>$\sigma_{S&amp;P}$ S&amp;P100 Daily Returns Volatility</td>
<td>±</td>
</tr>
<tr>
<td>$R_{M,t} - r_t$ Return on NYSE, AMEX and NASDAQ – 1M Treasury bill</td>
<td>-</td>
</tr>
<tr>
<td>DefSpread Moody’s BAA yield – Moody’s AAA yield</td>
<td>+</td>
</tr>
</tbody>
</table>

Table 3: Summary Statistics of EDPs

<table>
<thead>
<tr>
<th></th>
<th>Total Sample</th>
<th>All years</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>n-1</td>
<td>n-2</td>
</tr>
<tr>
<td>M Mean</td>
<td>0.57</td>
<td>0.35</td>
</tr>
<tr>
<td>LS Mean</td>
<td>1.23</td>
<td>0.27</td>
</tr>
<tr>
<td>ER Mean</td>
<td>1.11</td>
<td>0.74</td>
</tr>
<tr>
<td>M Standard Deviation</td>
<td>0.27</td>
<td>0.24</td>
</tr>
<tr>
<td>LS Standard Deviation</td>
<td>0.66</td>
<td>0.66</td>
</tr>
<tr>
<td>ER Standard Deviation</td>
<td>0.43</td>
<td>0.48</td>
</tr>
</tbody>
</table>
### Table 4: Cross-section Analysis EDPs and Volatility

<table>
<thead>
<tr>
<th>Model</th>
<th>a</th>
<th>t-stat</th>
<th>b</th>
<th>t-stat</th>
<th>c</th>
<th>t-stat</th>
<th>F</th>
<th>Adj R²</th>
</tr>
</thead>
<tbody>
<tr>
<td>Merton</td>
<td>-0.344</td>
<td>-10.9</td>
<td>0.704</td>
<td>16.3</td>
<td>0.26</td>
<td>10.9</td>
<td>150.2*</td>
<td>0.515</td>
</tr>
<tr>
<td>LS</td>
<td>-0.706</td>
<td>-8.3</td>
<td>1.601</td>
<td>13.8</td>
<td>0.344</td>
<td>5.4</td>
<td>95.5*</td>
<td>0.402</td>
</tr>
<tr>
<td>ER</td>
<td>-0.632</td>
<td>-9.9</td>
<td>1.474</td>
<td>16.9</td>
<td>0.333</td>
<td>6.9</td>
<td>143*</td>
<td>0.503</td>
</tr>
</tbody>
</table>

* Confidence level at 1%.

### Table 5: Cross-section Analysis of EDPs and Idiosyncratic Risk

<table>
<thead>
<tr>
<th>Model</th>
<th>a</th>
<th>t-stat</th>
<th>b</th>
<th>t-stat</th>
<th>c</th>
<th>t-stat</th>
<th>F</th>
<th>Adj R²</th>
</tr>
</thead>
<tbody>
<tr>
<td>Merton</td>
<td>-0.368</td>
<td>-18.1</td>
<td>0.263</td>
<td>8.7</td>
<td>0.543</td>
<td>23.7</td>
<td>471.7*</td>
<td>0.77</td>
</tr>
<tr>
<td>LS</td>
<td>-0.845</td>
<td>-13.1</td>
<td>0.871</td>
<td>9.1</td>
<td>1.001</td>
<td>13.8</td>
<td>218.8*</td>
<td>0.608</td>
</tr>
<tr>
<td>ER</td>
<td>-0.716</td>
<td>-15.1</td>
<td>0.837</td>
<td>11.9</td>
<td>0.834</td>
<td>15.7</td>
<td>314.3*</td>
<td>0.69</td>
</tr>
</tbody>
</table>

* Confidence level at 1%.

### Table 6: Cross-section Analysis of EDPs Volatility and Idiosyncratic Risk

<table>
<thead>
<tr>
<th>Model</th>
<th>a</th>
<th>t-stat</th>
<th>b</th>
<th>t-stat</th>
<th>c</th>
<th>t-stat</th>
<th>d</th>
<th>t-stat</th>
<th>F</th>
<th>Adj R²</th>
</tr>
</thead>
<tbody>
<tr>
<td>Merton</td>
<td>-0.415</td>
<td>-19.9</td>
<td>0.361</td>
<td>10.9</td>
<td>0.103</td>
<td>5.9</td>
<td>0.476</td>
<td>19.6</td>
<td>365.1*</td>
<td>0.795</td>
</tr>
<tr>
<td>LS</td>
<td>-0.854</td>
<td>-12.2</td>
<td>0.889</td>
<td>8</td>
<td>0.02</td>
<td>0.3</td>
<td>0.988</td>
<td>12.1</td>
<td>145.4*</td>
<td>0.607</td>
</tr>
<tr>
<td>ER</td>
<td>-0.749</td>
<td>-14.7</td>
<td>0.908</td>
<td>11.2</td>
<td>0.074</td>
<td>1.7</td>
<td>0.533</td>
<td>13.2</td>
<td>212.1*</td>
<td>0.693</td>
</tr>
</tbody>
</table>

* Confidence level at 1%.
### Table 7: Logistic Regressions

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
<th>(6)</th>
<th>(7)</th>
<th>(8)</th>
<th>(9)</th>
<th>(10)</th>
</tr>
</thead>
<tbody>
<tr>
<td>M</td>
<td>5.356</td>
<td></td>
<td></td>
<td>3.856</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.000)</td>
<td></td>
<td></td>
<td>(0.000)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>LS</td>
<td>1.839</td>
<td></td>
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<td></td>
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<td></td>
</tr>
<tr>
<td></td>
<td>(0.000)</td>
<td></td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>ER</td>
<td>2.701</td>
<td></td>
<td></td>
<td>1.542</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.000)</td>
<td></td>
<td></td>
<td>(0.011)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

### Idiosyncratic Risk

| Debt Ratio | 9.326 |        | 4.439 | 4.557 |          |        |      |        |          |          |
|            | (0.000)|        | (0.008)| (0.031)|          |        |      |        |          |          |
| MB Assets  | -1.160|        | -0.010| 0.291 |          |        |      |        |          |          |
| BM Equity  |       |        |       |       |          |        |      |        |          |          |
| Volatility | -0.411|        |       |       |          |        |      |        |          |          |
|            | (0.000)| (0.000)| (0.000)| (0.000)| (0.240) | (0.000)| (0.000)| (0.000)| (0.000) | (0.000) |
| Observations | 282  | 282  | 282  | 282  | 282  | 282  | 282  | 282  | 282  | 282  |
| -2 Log L   | 148.3 | 166.5 | 154.8 | 170.9 | 154.9 | 204.8 | 214.4 | 214.9 | 137.4 | 147.8  |
| Nagelkerke R² | 0.397 | 0.298 | 0.362 | 0.273 | 0.361 | 0.069 | 0.007 | 0.003 | 0.452 | 0.399  |

p-values in parentheses. The –2LogL statistic has a $\chi^2$ distribution with $n-q$ degrees of freedom, where $q$ is the number of parameters in the model. To all logistic regressions, we cannot reject the null hypothesis, which means that model fits the data (the $\chi^2$ statistics are corrected according to Shumway (2001) suggestions).

### Table 8: Factor Analysis

<table>
<thead>
<tr>
<th>Factors</th>
<th>Merton Model</th>
<th>LS Model</th>
<th>ER Model</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Eigenvalue</td>
<td>Cumulative %</td>
<td>Eigenvalue</td>
</tr>
<tr>
<td>1</td>
<td>7.6</td>
<td>32.8</td>
<td>7.1</td>
</tr>
<tr>
<td>2</td>
<td>5.8</td>
<td>57.9</td>
<td>4.9</td>
</tr>
<tr>
<td>3</td>
<td>2.0</td>
<td>66.5</td>
<td>2.8</td>
</tr>
<tr>
<td>4</td>
<td>1.9</td>
<td>74.8</td>
<td>2.4</td>
</tr>
<tr>
<td>5</td>
<td>1.1</td>
<td>79.5</td>
<td>1.5</td>
</tr>
<tr>
<td>6</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>RMSR</td>
<td>0.057</td>
<td>0.039</td>
<td>0.048</td>
</tr>
<tr>
<td>Nonredundant Residuals</td>
<td>7.1%</td>
<td>1.2%</td>
<td>3.2%</td>
</tr>
</tbody>
</table>

Root Mean Square Residual (RMSR) = $\sqrt{\frac{\sum \sum res^2_{ij}}{p(n-q)}}$, where $p$ is the number of companies and $res$ gives the amount of correlation that is not explained by the retained factors. Nonredundant residuals are computed as a percentage of the number of nonredundant residuals with absolute values greater than 0.10.
Table 9: Principal Components

<table>
<thead>
<tr>
<th>PCA Factors</th>
<th>Merton Model</th>
<th>LS Model</th>
<th>ER Model</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Eigenvalue</td>
<td>% of variance</td>
<td>Eigenvalue</td>
</tr>
<tr>
<td>1</td>
<td>2.7</td>
<td>55.0</td>
<td>2.9</td>
</tr>
<tr>
<td>2</td>
<td>1.7</td>
<td>34.3</td>
<td>2.1</td>
</tr>
</tbody>
</table>

Cum. % explained by PCA 89.3 88.4 91.2

Table 10: Determinants of Default Correlation Factors

For each model and each default correlation factor, 1 or 2, we estimate the following regression: \( \text{Fact}_{t-1} = \alpha + \beta_1 r_{t1}^{10} + \beta_2 (r_{t1}^{10})^2 + \beta_3 (r_{t1}^{10} - r_1^2) + \beta_4 \sigma_{S&P, t} + \beta_5 (R_M, t - r_{t1m}) + \beta_6 \text{DefSpread,} \) using a stepwise procedure. Beneath the variables, in parenthesis, we report significance values. n. e. means not entered in the regression.

<table>
<thead>
<tr>
<th></th>
<th>Merton Model</th>
<th>LS Model</th>
<th>ER Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Intercept</td>
<td>1.54</td>
<td>-0.81</td>
<td>-0.88</td>
</tr>
<tr>
<td></td>
<td>(0.017)</td>
<td>(0.027)</td>
<td>(0.011)</td>
</tr>
<tr>
<td>( r_{t1}^{10} )</td>
<td>n. e.</td>
<td>n. e.</td>
<td>n. e.</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( (r_{t1}^{10})^2 )</td>
<td>n. e.</td>
<td>n. e.</td>
<td>n. e.</td>
</tr>
<tr>
<td>( r_{t1}^{10} - r_1^2 )</td>
<td>n. e.</td>
<td>80.78</td>
<td>88.10</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.007)</td>
<td>(0.002)</td>
</tr>
<tr>
<td>( \sigma_{S&amp;P} )</td>
<td>-8.00</td>
<td>n. e.</td>
<td>n. e.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.030)</td>
<td></td>
</tr>
<tr>
<td>( R_M - r_{t1m} )</td>
<td>-2.59</td>
<td>n. e.</td>
<td>n. e.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.025)</td>
<td></td>
</tr>
<tr>
<td>( \text{DefSpread} )</td>
<td>n. e.</td>
<td>n. e.</td>
<td>n. e.</td>
</tr>
<tr>
<td>Adj. R^2</td>
<td>0.56</td>
<td>0.43</td>
<td>0.52</td>
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<tr>
<td></td>
<td>(0.005)</td>
<td>(0.007)</td>
<td>(0.002)</td>
</tr>
</tbody>
</table>
Figure 1 - EDPs of Bankrupt Companies

Figure 2 – Distribution of EDPs: Merton’s Model
Figure 3 – Distribution of EDPs: Longstaff and Schwartz Model

Figure 4 – Distribution of EDPs: Ericsson and Reneby Model