Intra-Household Allocation and Consumption of WIC-Approved Foods: A Bayesian Approach

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Abstract

WIC, the Special Supplemental Nutrition Program for Women, Infants, and Children, is a widely-studied public food assistance program that aims to provide foods, nutrition education and other services to at-risk, low income children and pregnant/breastfeeding mothers. From a policy perspective, it is of interest to assess the efficacy of the WIC program - how much, if at all, does the program improve the nutritional outcomes of WIC families? In this paper we address two important issues related to the WIC program that have not been extensively addressed in the past. First, although the WIC program is primarily devised with the intent of improving the nutrition of “target” children and mothers, it is possible that WIC may also change the consumption of foods by non-targeted individuals within the household. Second, although WIC eligibility status is predetermined, participation in the program is voluntary and therefore potentially endogenous. We make use of a triangular treatment-response model in which the dependent variable is the requirement-adjusted calcium intake from milk consumption and the endogenous variable is WIC participation, and estimate it using Bayesian methods. Using data from the CSFII 1994-1996, we find that the correlation between the errors of our two equation system is strong and positive, suggesting that families participating in WIC have an unobserved propensity for high calcium consumption. The direct “structural” WIC parameters, however, do not support the idea that WIC participation leads to increased levels of calcium consumption from milk.

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1 Introduction

In fiscal year 2006, the USDA spent nearly $53 billion on food assistance programs (USDA/ERS 2007). The third largest of these programs, the Special Supplemental Nutrition Program for Women, Infants, and Children (commonly and henceforth denoted as WIC), has been widely studied in the health and nutrition literatures, and aims to serve the public by providing supplemental foods, nutrition education and other services to foster the growth, development and long-term health of participating children.

For families that qualify for WIC participation, the program provides access to nutritious foods to supplement the diets of infants, children up to age five, and pregnant, breastfeeding and postpartum women. The program benefits, usually in the form of checks or vouchers, allow participants to obtain specific "packages" of foods. These foods include infant formula, milk, cheese, eggs, juice, cereals, peanut butter/dried beans, and, for fully breast-feeding mothers, these also include carrots and tuna.

From a policy perspective, it is of primary interest to assess the efficacy of the WIC program - how much, if at all, does the program improve the nutritional outcomes of WIC families? In this paper we employ a Bayesian methodology to address this question and estimate the impact of WIC participation on a specific nutritional outcome - calcium intake via milk consumption. Our study is certainly not the first in this regard, as other efforts using different models and maintained assumptions have been conducted in the past. For example, Oliveira and Chandran (2005) find that participation in the WIC program increases the consumption for some types of WIC-approved foods for WIC children compared to eligible nonparticipating children and children living in households with income too high to be eligible for WIC (income greater than 185% of the poverty threshold). Other efforts in this regard include the studies of Rose, Habicht and Devaney (1998), Burstein, et al. (2000), Oliveira and Gundersen, (2000) Ponza, et al. (2004) and Siega-Riz, et al. (2004) who generally find positive impacts associated with the WIC program.

There are, however, two important issues related to the WIC program that have not been extensively addressed in past work, and we seek to address these issues in the current paper. First, although the WIC program is primarily devised with the intent of improving the nutrition of “target” children and mothers, it is possible that WIC may also change the consumption of foods by non-targeted individuals within the household. This has been referred to as “spillover” (Oliveira and Chandran 2005) or “leakage” (Barrett 2002) of WIC benefits. As Oliveira and Chandran note, this might be done if 1) receipt of WIC benefits frees up food dollars for use to benefit other, nonparticipating children; 2) nutrition education changes food selection for all members; or 3) WIC foods are shared with non-WIC household members. Little is known about the degree to which this occurs. In the current paper, we formally address this issue by comparing the impact of WIC participation on both target household members as well as non-target members of WIC families.
Second, the previous literature on this topic has certainly been aware of the potential endogeneity of WIC participation, and in some cases, has interpreted the obtained results with caution in light of this concern. To our knowledge, however, no studies in the literature have dealt with this problem extensively. To this end, we make use of a triangular treatment-response model in which the dependent variable is the requirement-adjusted calcium intake from milk consumption and the (potentially) endogenous variable is WIC participation. Ostensibly, WIC participation will lead to increased calcium consumption through milk, though in the presence of endogenous participation, this need not be the case. For example, families who choose to participate in WIC may simultaneously (and unobservably) be quite concerned regarding the nutritional intake of each family member, and thus those participating in WIC may have high calcium consumption even in the absence of WIC. Moreover, the presence of WIC may enable families to consume calcium through other sources, so that WIC would actually lead to a reduction in calcium intake through milk.

In terms of our posterior predictive distributions, we find results consistent with our prior expectations and the majority of past work on this topic. That is, WIC target individuals have higher levels of calcium intake than their non-WIC counterparts. However, the posterior predictives combine two sources of information: what we might term the “structural” effect of WIC participation as well as an unobserved correlation between the errors of the participation and outcome equations. As one might suspect, we find that the correlation between errors of our two equation system is strong and positive, suggesting that families participating in WIC have an unobserved propensity for high calcium consumption. What drives the intuitive ordering among the posterior predictives is primarily the selection effect - those families in WIC would have had large levels of calcium consumption in the absence of the program. The direct “structural” WIC parameters do not directly support the idea that WIC participation leads to increased levels of calcium consumption, a finding that is, to our knowledge, somewhat new to this literature. Indeed, these families may be substituting away from milk and toward other more “luxury” alternatives, a finding that has significant implications for the selection of foods within the WIC program.

The paper proceeds as follows. The next section describes the model specification and the associated Bayesian posterior simulator. The data used in the analysis are described in section 3, followed by the empirical results in section 4. The paper concludes with a summary of findings in section 5.

2 The Model, Posterior Simulator and Posterior Predictives

We first let $w_h$ be a binary variable equal to one if household $h$ participates in WIC and equals zero otherwise. Within a given household, some members, including children under 5 and pregnant/breastfeeding mothers, will be target individuals, i.e., those family members the WIC program is primarily designed to serve. To this end, we will let $T_{ih}$ be an exogenous binary variable denoting
if individual $i$ in household $h$ is a WIC targeted individual. The construction of these two variables leads to the categorization of all individuals in our sample into four mutually exclusive groups:

$$G_{1,ih} = w_h * T_{ih} \text{ (targeted individual in a WIC participating household)},$$
$$G_{2,ih} = w_h * (1 - T_{ih}) \text{ (non-targeted individual in a WIC participating household)},$$
$$G_{3,ih} = (1 - w_h) * T_{ih} \text{ (targeted individual in a WIC eligible but non-participating household)},$$
$$G_{4,ih} = (1 - w_h) * (1 - T_{ih}) \text{ (non-targeted individual in a WIC eligible but non-participating household)}.$$

Our outcome variable of interest is requirement-adjusted calcium intake through milk consumption. We represent this variable as $c_{ih}$. Importantly, there is a censoring problem associated with calcium intake in our data, since approximately 16% of our sample has identically zero consumption values. To this end, we follow Chib (1992) and Albert and Chib (1993) and work with latent consumption $c_{ih}^*$, which is assumed to be generated by:

$$c_{ih}^* = x_{ih} \alpha + \epsilon_{ih}, \quad (1)$$

and

$$c_{ih} = \max\{0, c_{ih}^*\}. \quad (2)$$

The group identifiers $G_1 \rightarrow G_4$ above together with other relevant demographic variables such as age, income, gender indicators, etc. are included in the vector $x_{ih}$. By comparing the $\alpha$ coefficients across these four groups, we can determine if WIC participation has an important effect on calcium consumption, and moreover, we can test for the presence of the hypothesized “spillover” effects within a WIC household. That is, we can determine whether or not non-target members in WIC households have higher levels of calcium intake through milk consumption than non-target members of non-WIC households.

As stressed in the introduction of this paper, WIC participation is voluntary, and thus the binary indicator $w_h$ (and associated group identifiers $G_1 \rightarrow G_4$) are not necessarily exogenous in (1). That is, household heads choosing to participate in WIC could, for example, be very concerned about the nutritional intakes of its constituents, thus leading to higher levels of calcium consumption for these families on average. To this end, we first consider the household-level decision to participate in WIC:

$$w_h^* = z_h \beta + \nu_h, \quad \nu_h \sim N(0, 1) \quad (3)$$

where

$$w_h = \begin{cases} 1 & \text{if } w_h^* > 0 \\ 0 & \text{if } w_h^* \leq 0 \end{cases} \quad (4)$$

and $z_h$ is a vector of household specific characteristics.

\textsuperscript{1}We follow standard conventions of using capital letters to denote matrix quantities and bold script to denote vectors or matricies.
To account for the potential endogeneity of WIC participation, we allow the errors of (1) and (3) to be correlated. That is, household level unobservables that make a family more likely to participate in WIC may also make that family more likely to have high levels of calcium consumption. We choose to accommodate this type of correlation by including a household-specific error term in (1) and allowing this error to be correlated with $\nu_h$ in (3). The intuition behind this modeling assumption is that a household head who chooses to participate in WIC will also tend to guide meal preparation in the household and monitor the nutritional habits of the household members. Thus, unobservable factors affecting WIC participation will likely spill over and correlate with the nutritional intakes of all the family members, and should probably correlated in a similar way across each member. To this end, we consider the following model:

$$c_{ih}^* = x_{ih}\alpha + \psi s_{ih}^* + u_h + \epsilon_{ih}, \quad (5)$$

$$w_h^* = z_h\beta + \nu_h, \quad (6)$$

where

$$\begin{bmatrix} u_h \\ \nu_h \end{bmatrix} \sim N \left( \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \sigma_u^2 & \sigma_{uv} \\ \sigma_{uv} & 1 \end{pmatrix} \right), \quad (7)$$

and

$$\epsilon_{ih} | x, z, s^* \sim N(0, \sigma_\epsilon^2). \quad (8)$$

Equations (5) and (6) now represent a standard two equation treatment-response model using only observed rather than potential outcomes.\(^2\) However, we note that equation (5), unlike its counterpart in (1), has included a latent variable $s_{ih}^*$. This latent variable is included, like the model of Chen, Dey and Shao (1999), to capture possible skew in the distribution of calcium consumption.\(^3\) These latent variables are specified to be generated from a known distribution with one-sided support, thereby introducing the possibility of accommodating skew in the outcome distribution beyond what is implied by normality (given that $c_{ih}^* > 0$). A rather standard choice in this regard, as employed in Chen, Dey and Shao (1999), is to assume that $s_{ih}^*$ is generated from a half-normal distribution,

$$s_{ih}^* | x, z \sim TN_{(0,\infty)}(0,1),$$

with $TN_{(a,b)}(\mu,\sigma^2)$ denoting a normal distribution with mean $\mu$ and variance $\sigma^2$ truncated to the interval $(a,b)$. When integrating the conditional density for $c_{ih}^*$ (given $s_{ih}^*$) over this half-normal for $s_{ih}^*$, it can be shown that, marginally, $c_{ih}^*$ will have a skew-normal distribution [e.g., Azzalini and Dalla Valle (1996), Chen, Dey and Shao (1999) and Branco and Dey (2002)]. The sign of the parameter $\psi$ governs the direction of the skew (i.e., positive values produce a distribution with a right-skew,


\(^3\)Note that, unlike adopting the log specification, the model in (5) introduces skew without having to address potential issues such as taking the log of negative values (and simultaneously considering the mass point at zero consumption), or introducing an additional “hurdle” or “threshold” to the analysis. This representation is, of course, not as flexible as other alternatives such as Gaussian mixtures or Dirichlet processes, but is a simpler alternative that may be adequately flexible to capture the salient features of a given problem.
conversely for negative values of $\psi$, and $\psi = 0$ reduces to joint normality). Since the potential for such skew exists in both the conditional and unconditional distributions of calcium consumption, we adopt the above procedure for handling this issue. As shown in our empirical results section, the data strongly support the hypothesis of $\psi \neq 0$ so that the default assumption of joint normality is not appropriate for this data.

With the above formulation, the composite error term $\psi s^*_h + u_h + \epsilon_ih$ is not mean zero since $s^*_h$ is not mean zero. Though this shift will be “absorbed” by the intercept parameter, this creates a muddled interpretation of the parameter $\psi$ and may lead to slower mixing of the posterior simulations.\(^4\) We handle this issue by simply shifting the distribution of $s^*_h$ back by its mean, $\sqrt{(2/\pi)}$. Thus, in our analysis, we specify\(^5\)

$$
\begin{align*}
s^*_h | x, z & \sim TN\left(-\sqrt{2/\pi}, \infty\right)\left(-\sqrt{2/\pi}, 1\right). \\
\end{align*}
$$

and the model is given by (5)-(8) together with the revised distributional assumption on $s^*_h$ given in (9).

\section{2.1 The Joint Posterior}

For the implementation of the posterior simulator, it will be instructive to work with the population expectation of $u_h$ given $\nu_h$. Given the joint Normality assumption above, we can write

$$
u_h = \sigma_u \nu_h + \eta_h,$$

where

$$
\eta_h \overset{iid}{\sim} N(0, \sigma^2_\eta), \quad \text{and} \quad \sigma^2_\eta \equiv \sigma^2_u - \sigma^2_{uv}.
$$

Thus, we can re-write our initial equation system in the following way:

$$
\begin{align*}
\epsilon^*_i h &= x_ih \alpha + \psi s^*_i h + \sigma_{uv} \nu_h + \eta_h + \epsilon_i h \\
\nu^*_h &= z_h \beta + \nu_h
\end{align*}
$$

where

$$
\begin{align*}
\epsilon_i h & \overset{iid}{\sim} N(0, \sigma^2_\epsilon) \\
\nu_h & \overset{iid}{\sim} N(0, 1) \\
\eta_h & \overset{iid}{\sim} N(0, \sigma^2_\eta).
\end{align*}
$$

Thus, conditioned on the common $\nu_h$, the consumption and WIC participation equations are independent.

\(^4\)This issue has been pointed out by Pewsey (2000) and others.

\(^5\)In generated data experiments, this simple transformation seemed to improve the mixing of the posterior simulations.
Let

$$\delta = [\alpha', \beta', \psi, \sigma_{uv}^2, \sigma_{\eta}^2]'$$

denote the parameters of our model other than the random effects \( \eta \). In addition, let \( n_h \) denote the number of individuals in household \( h \), \( H \) denote the total number of households in the sample, \( NH = \sum_{h=1}^{H} n_h \), and finally, define

$$c^*_h = \begin{bmatrix} c^*_{1h} \\ c^*_{2h} \\ \vdots \\ c^*_{n_{h},h} \end{bmatrix}, \quad X_h = \begin{bmatrix} x_{1h} \\ x_{2h} \\ \vdots \\ x_{n_{h}h} \end{bmatrix}, \quad s^*_h = \begin{bmatrix} s^*_{1h} \\ s^*_{2h} \\ \vdots \\ s^*_{n_{h}h} \end{bmatrix},$$

$$c^* = \begin{bmatrix} c^*_1 \\ c^*_2 \\ \vdots \\ c^*_H \end{bmatrix}, \quad s^* = \begin{bmatrix} s^*_1 \\ s^*_2 \\ \vdots \\ s^*_H \end{bmatrix}, \quad w^* = \begin{bmatrix} w^*_1 \\ w^*_2 \\ \vdots \\ w^*_H \end{bmatrix}, \quad \eta = \begin{bmatrix} \eta_1 \\ \eta_2 \\ \vdots \\ \eta_H \end{bmatrix}.$$

As in Albert and Chib (1993), we will include the latent \( c^*, w^*, s^* \) and vector of random effects \( \eta \) into our posterior, and thus work with an augmented posterior of the form

$$p(c^*, w^*, s^*, \delta, \eta|c, w) \propto p(c, w|c^*, w^*, s^*, \delta, \eta)p(c^*, w^*, s^*|\delta, \eta)p(\eta|\delta)p(\delta)$$

$$= p(\delta) \prod_{h=1}^{H} \left[ p(w_h|w_h^*)p(c^*_h, w_h^*, s_h^*, \delta, \eta_h)p(\eta_h|\delta) \left( \prod_{i < h} p(c_{ih}|c_{ih})p(s_{ih}^*) \right) \right].$$

In the first line above, we write the posterior as proportional to the full joint distribution (of parameters, latent and observed data), and decompose this joint distribution into a sequence of conditionals times marginals. The densities \( p(\eta|\delta) \) and \( p(\delta) \) denote prior distributions for these parameters, and, in the second line of the above, we incorporate the assumed (conditional) independence across households. Finally, in regard to the density \( p(c, w|c^*, w^*, s^*, \delta, \eta) \), we note that the distribution for \( w_h \) depends only on \( w_h^* \) (and is degenerate given its value), and likewise, the distribution of \( c_{ih} \) depends only on \( c_{ih}^* \) (and is degenerate given its value). That is,

$$p(w_h|w_h^*) = I(w_h = 1)I(w_h^* > 0) + I(w_h = 0)I(w_h^* \leq 0)$$

and

$$p(c_{ih}|c_{ih}^*) = I(c_{ih} = c_{ih}^*)I(c_{ih}^* > 0) + I(c_{ih} = 0)I(c_{ih}^* \leq 0).$$

As for the joint distribution of \( c^*_h \) and \( w^*_h \), note that

$$\begin{bmatrix} c^*_h \\ w^*_h \end{bmatrix} | s^*_h, \delta, \eta_h \overset{\text{ind}}{\sim} N \left( \begin{bmatrix} x_h \alpha + \psi s^*_h + \eta_h \epsilon_n \end{bmatrix}, \begin{bmatrix} \sigma^2 \beta + \sigma_{uv} \epsilon_n \beta' \epsilon_n & \sigma_{uv} \epsilon_n \beta' \epsilon_n \\ \sigma_{uv} \epsilon_n \beta' \epsilon_n & \sigma_{uv} \epsilon_n \beta' \epsilon_n \end{bmatrix} \right),$$

where \( \epsilon_n \) is a \( n_h \times 1 \) vector of ones, and likewise \( \beta \) is the identity matrix of dimension \( n_h \).
2.2 The posterior simulator

We fit this model using recent advances in Markov Chain Monte Carlo (MCMC) techniques, namely the Gibbs sampler [e.g., Gelfand et al. (1990), Casella and George (1992), Albert and Chib (1993)]. Implementation of the Gibbs sampler involves deriving and then iteratively simulating from the conditional posterior distributions of the model’s parameters. The sequence of simulations produced from this sampling procedure forms a Markov chain that, under certain regularity conditions, converges to the target distribution (i.e., the joint posterior). To mitigate the effect of initial conditions on this chain, an initial set of pre-convergence or “burn-in” simulations are discarded, and the remaining set of simulations are then used to calculate posterior features of interest.

Before describing the specific steps of our posterior simulator, we must first introduce our priors. We let

\[
\gamma = \begin{bmatrix}
\alpha \\
\psi \\
\beta
\end{bmatrix}
\]

and specify priors of the form

\[
\begin{align*}
\gamma &\sim N(\mu_\gamma, V_\gamma) \quad (10) \\
\sigma_{uv} &\sim N(\mu_{uv}, V_{uv}) \quad (11) \\
\sigma_\epsilon^2 &\sim IG(a_\epsilon, b_\epsilon) \quad (12) \\
\sigma_\eta^2 &\sim IG(a_\eta, b_\eta). \quad (13)
\end{align*}
\]

Our complete Gibbs algorithm consists of 8 steps, and the first two of these form a blocking step [e.g., Chib and Carlin (1999)], where the parameters \( \gamma = [\alpha' \ \psi' \ \beta']' \) and random effects \( \eta \) are sampled in a single block. We do this via the method of composition. That is, we first sample \( \gamma \) from its conditional posterior, where the random effects \( \eta \) have been integrated out. We then sample \( \eta \) by drawing each \( \eta_h \) independently from its complete conditional posterior. For simplicity in notation below, we let \( \Gamma = [\delta' \ \epsilon' \ \omega' \ \beta']' \) and let \( \Gamma_{-x} \) denote all parameters other than \( x \).

**Step 1:** \( \gamma|\Gamma_{-\gamma}, c, w \).

First, define

\[
\begin{bmatrix}
X_h & s_h^* & 0 \\
0 & 0 & z_h
\end{bmatrix}, \quad \begin{bmatrix}
c_h^* \\
w_h^*
\end{bmatrix}
\]

and

\[
\Sigma_h = \begin{bmatrix}
\sigma_\epsilon^2 I_{nh} + (\sigma_\eta^2 + \sigma_{uv}^2) \ell_{nh}^' \ell_{nh} & \sigma_{uv} \ell_{nh}^' \\
\sigma_{uv} \ell_{nh} & 1
\end{bmatrix}.
\]

It follows that

\[
\gamma|\Gamma_{-\gamma}, c, w \sim N(D_\gamma, D_\gamma), \quad (14)
\]
where

\[
D_{\gamma} = \left[ \sum_h (X_h' \Sigma_h^{-1} X_h) + V_\gamma^{-1} \right]^{-1}, \quad \text{and} \quad d_{\gamma} = \sum_h (X_h' \Sigma_h^{-1} e_h^* + V_\gamma^{-1} \mu_{\gamma}).
\]

**Step 2:** \(\eta_h | \Gamma_{-\eta_h}, c, w\)

\[
\eta_h | \Gamma_{-\eta_h}, c, w \overset{\text{ind}}{\sim} N(D_{\eta_h} d_{\eta_h}, D_{\eta_h}), \quad h = 1, 2, \ldots, H,
\]

where

\[
D_{\eta_h} = \frac{\sigma^2_{\eta}}{n_h \sigma^2_{\eta} + \sigma^2_{\epsilon}}
\]

\[
d_{\eta_h} = \frac{1}{\sigma^2_{\epsilon}} \sum_{i \in h} (c_{ih}^* - x_{ih} \alpha - s_{ih}^* \psi - \sigma_{uv}[w_h^* - z_h \beta]).
\]

**Step 3:** \(\sigma_{uv} | \Gamma_{-\sigma_{uv}}, c, w\)

First, define the \(NH \times 1\) vectors \(V\) and \(\eta\) as follows:

\[
V \equiv \begin{bmatrix}
\ell_{n1}[w_1^* - z_1 \beta] \\
\ell_{n2}[w_2^* - z_2 \beta] \\
\vdots \\
\ell_{nH}[w_H^* - z_H \beta]
\end{bmatrix}, \quad \eta \equiv \begin{bmatrix}
\ell_{n1}[\eta_1] \\
\ell_{n2}[\eta_2] \\
\vdots \\
\ell_{nH}[\eta_H]
\end{bmatrix}.
\]

It follows that

\[
\sigma_{uv} | \Gamma_{-\sigma_{uv}}, c, w \sim N(D_{\sigma_{uv}} d_{\sigma_{uv}}, D_{\sigma_{uv}}),
\]

where

\[
D_{\sigma_{uv}} = (V'V/\sigma^2_{\epsilon} + V_{uv}^{-1})^{-1}, \quad d_{\sigma_{uv}} = V'(c^* - X \alpha - s^* \psi - \eta)/\sigma^2_{\epsilon} + V_{uv}^{-1} \mu_{\sigma_{uv}}.
\]

**Step 4:** \(\sigma^2_{\epsilon} | \Gamma_{-\sigma^2_{\epsilon}}, c, w\)

\[
\sigma^2_{\epsilon} | \Gamma_{-\sigma^2_{\epsilon}}, c, w \sim IG \left( \frac{NH}{2} + a_{\epsilon}, \left[ b_{\epsilon}^{-1} + \frac{1}{2} \sum_{i=1}^{NH} (c_{ih}^* - x_{ih} \alpha - s_{ih}^* \psi - \eta_h - \sigma_{uv}[w_h^* - z_h \beta])^2 \right]^{-1} \right).
\]

**Step 5:** \(\sigma^2_{\eta} | \Gamma_{-\sigma^2_{\eta}}, c, w\)

\[
\sigma^2_{\eta} | \Gamma_{-\sigma^2_{\eta}}, c, w \sim IG \left( \frac{H}{2} + a_{\eta}, \left[ b_{\eta}^{-1} + \frac{1}{2} \sum_{h=1}^{H} (\eta^2_h) \right]^{-1} \right).
\]

**Step 6:** \(w^* | \Gamma_{-w^*}, c, w\)
Each of the latent variables in the WIC participation equation are sampled independently as follows:

\[ w^*_h | \Gamma_{-w^*}, c, w \sim \begin{cases} 
    TN_{(0, \infty)}(\mu_{w^*_h}, \sigma_{w^*_h}^2) & \text{if } w_h = 1 \\
    TN_{(-\infty, 0)}(\mu_{w^*_h}, \sigma_{w^*_h}^2) & \text{if } w_h = 0 
\end{cases} \]  

(19)

where

\[ \mu_{w^*_h} = z_h \beta + \sigma_{uv}^2 \Psi'_{n_h} \left[ \sigma^2 \Gamma_{n_h} + \sigma_{uv}^2 \Psi'_{n_h} \right]^{-1} \left( c^*_h - X_h \alpha - s^*_h \Psi - \eta \Psi_{n_h} \right), \]

\[ \sigma_{w^*_h}^2 = 1 - \sigma_{uv}^2 \Psi'_{n_h} \left[ \sigma^2 \Gamma_{n_h} + \sigma_{uv}^2 \Psi'_{n_h} \right]^{-1} \Psi_{n_h}. \]

**Step 7: c^*_i | \Gamma_{-c^*}, c, w**

Note that, conditioned on \( \eta_h \) and the remaining parameters of the model, each latent \( c^*_i \) can be sampled independently from its conditional posterior:

\[ c^*_i | \Gamma_{-c^*_i}, c, w \sim TN_{(-\infty, 0)}(\mu_{c^*_i}, \sigma_{c^*_i}^2) \quad \text{if } c_i = 0, \]

(20)

where

\[ \mu_{c^*_i} = x_i \alpha + s^*_i \Psi + \eta_i + \sigma_{uv}(w^*_i - z_i \beta). \]

When \( c_i > 0 \), the conditional posterior for \( c^*_i \) is degenerate around the observed \( c_i \) and does not need to be simulated.

**Step 8: s^*_i | \Gamma_{-s^*_i}, c, w**

The assumptions of our model imply that each \( s^*_i \) can be sampled independently from their complete conditional posteriors. Completing the square in \( s^*_i \) yields a posterior conditional of the form:

\[ s^*_i | \Gamma_{-s^*_i}, c, w \overset{\text{ind}}{\sim} TN_{(-\sqrt{2/\pi}, \infty)}(\mu_{s^*_i}, \sigma_{s^*_i}^2), \quad i = 1, 2, \ldots, NH, \]

(21)

where

\[ \mu_{s^*_i} = \frac{\psi(c^*_i - x_i \alpha - \eta_i - \sigma_{uv}(w^*_i - z_i \beta)) - \sqrt{2/\pi} \sigma_{s^*_i}^2}{\sigma_{s^*_i}^2 + \psi^2}, \]

and

\[ \sigma_{s^*_i}^2 = \frac{\sigma_{s^*_i}^2}{\sigma_{s^*_i}^2 + \psi^2}. \]

A Gibbs sampler proceeds by iteratively sampling from (14)-(21).

### 2.3 Posterior Predictive Intake Distribution

In our empirical application, we are primarily concerned with the calculation and comparison of intake distributions for individuals in each of the four groups of interest. To this end, we focus
on posterior prediction, and fix the exogenous covariates’ values for simplicity. Given our model, the posterior predictive intake distribution for such a representative agent with fixed covariates and $w_h = 1$, conditioned on the model parameters $\Gamma$, is given as

$$p(c_{ih}^*|w_h = 1, \Gamma) = p(c_{ih}^*|w_h^* > 0, \Gamma) = \left[ \Pr (w_h^* > 0|\Gamma) \right]^{-1} \int_0^\infty p(c_{ih}^*, w_h^*|\Gamma) \, dw_h^*. $$

After some manageable algebra, we perform the required integration and obtain:

$$p(c_{ih}^*|w_h = 1, \Gamma) = \Phi \left[ \frac{z_h \beta + [\sigma_{uv}/(\sigma_{uv}^2 + \sigma^2)] \left( c_{ih}^* - x_{ih} \alpha - \psi s_{ih}^* - \eta_h \right)}{\sqrt{\sigma^2 / [\sigma^2 + \sigma_{uv}^2]}} \right] \times \phi \left( c_{ih}^*; x_{ih} \alpha + \psi s_{ih}^* + \eta_h, \sigma^2 + \sigma_{uv}^2 \right) \Phi \left( z_h \beta \right). $$

The density in (22) is not of an immediately recognizable form, though the steps leading to its derivation suggest a method of obtaining draws directly from this density. Specifically, draws from (22) can be obtained from the following procedure:

First, sample

$$y_{ih}^* \sim TN(-z_h \beta, \infty) (0, 1).$$

Then, set

$$r_{ih}^* = \pi_0 + \pi_1 y_{ih}^* + \pi_2 \epsilon$$

where

$$\epsilon \sim N(0, 1)$$

$$\pi_0 = x_{ih} \alpha + \psi s_{ih}^* + \eta_h,$$

$$\pi_1 = \sigma_{uv},$$

$$\pi_2 = \sigma_{\epsilon}.$$ 

It can be shown that $r_{ih}^*$ has the density given in (22). The proof of this fact is reasonably straightforward, noting that $p(r_{ih}^*) = \int p(r_{ih}^*, y_{ih}^*) p(y_{ih}^*) \, dy_{ih}^*$ and subbing in the formulas above to perform the necessary integration.

Since this procedure obtains a draw from the posterior predictive for a given vector of parameters $\Gamma$, their influence can be marginalized out of the predictive by noting:

$$p(c_{ih}^*|w_h = 1, c, w) = \int p(c_{ih}^*|w_h = 1, \Gamma) p(\Gamma|c, w) \, d\Gamma. $$

Thus, for every post-convergence $\Gamma$ draw produced from the simulator, we apply the above procedure to obtain a draw from the posterior predictive. Though the details are suppressed here, similar steps
can be used to obtain the posterior predictive associated with the event that \( w_h = 0 \). Finally, calcium intake is linked to the latent \( c_{ih}^* \) by noting: \( c_{ih} = \max\{0, c_{ih}^*\} \), which is calculated for each iteration of the sampler.

### 3 The Data

Our empirical analysis makes use of data from the USDA 1994-96 Continuing Survey of Food Intake by Individuals (CSFII). The CSFII is a nationally representative, cross-sectional survey of individuals in households in the United States. The survey uses a randomization strategy to select certain members of the household to participate in a complete food intake survey; thus, not all members of a WIC household are present in our sample. For each of the sampled individuals, questions involving a twenty four hour recall of food intake were conducted on two nonconsecutive days. Importantly for our purposes, respondents report milk consumption and the consumption of milk-containing foods during this period.

Household and individual characteristics can be used to identify WIC eligible households, and we focus only on those individuals and households that are WIC eligible in our analysis. To be eligible for WIC, at least one individual in the household must be in a WIC-qualifying population group (women who are pregnant; non-breastfeeding women up to 6 months postpartum; breastfeeding women up to 1 year postpartum; infants under 1 year of age; or children from 1 year old up to his/her 5th birthday). The household’s income must also be at or below 185 percent of the Federal poverty guidelines, or the household must participate in other qualifying programs (Medicaid, Food Stamps, Temporary Assistance for Needy Families (TANF)). Finally, individual applicants must be at nutritional risk, as determined by a health professional. Although it is not possible to determine individuals that are at nutritional risk from the CSFII data, nearly all U.S. women and children meet this criterion (IOM 2002) so that, in practice, this additional constraint can be assumed to apply to all eligible individuals. Finally, we follow Oliveira and Chandran (2005) and define eligible households as those with incomes within 200 percent of the federal poverty guidelines.

Our final sample consists of 2,372 individuals from 1,036 households. As discussed in the previous section, these individuals were assigned into one of the four mutually exclusive groups (Table 1). For our analysis we define WIC targeted individuals as children of ages one through four and pregnant, lactating or breastfeeding women, and non-targeted individuals as children or adults in the household aged five and older. All households in our final sample are identified as WIC eligible by meeting the income criterion and having at least one targeted individual living in the household.

\[\text{Infants of age less than 1 year old are not included in the analysis because of their unique dietary requirements and intakes.}\]
Each of the four population groups described in Table 1 may have both adults and children. In order to compare the food intakes of individuals of varying age and gender, we convert the dependent variable, amount of milk consumed (grams), to a calcium-equivalent measure and then normalize the consumption in terms of the individuals' dietary requirement for calcium. This is accomplished in several steps. First, the CSFII 94-96 data set contains information on grams of milk consumed as a single food or an ingredient in a food containing dairy products. However, milk is commonly included as an ingredient in other non-dairy foods, and it is important to capture this aspect of calcium consumption in the construction of our dependent variable. To this end, we consult the Pyramid Servings Database for USDA Survey Food Codes, Version 2.0, which provides information on the amount of milk per 100 grams contained within a variety of different foods. The amounts of milk and milk ingredients from all foods consumed by an individual are then added together to produce the total amount of calcium consumed by the individual.

The reference amount of calcium required by individuals is expressed as Dietary Reference Intake (DRI) values (Dietary Reference Intakes, IOM 1997). The calcium requirement for children of ages one through three (500mg of calcium/day) was used as the base or reference amount to normalize consumption by other population groups. That is, the calcium intake of the surveyed individuals was converted into an age and gender equivalent measure. Thus, the dependent variable is measured as a requirement-adjusted amount of calcium (mg) received from the foods consumed. For example, if a young child reported an intake of 600mg per day of calcium, their reported intake of 600mg would be measured relative to their DRI (500mg) and converted to a 500mg reference value 600mg: 

\[\text{requirement-adjusted intake} = \frac{600\text{mg}}{500\text{mg}} \times 500\text{mg}.\]

For an adult with a DRI of 1000mg, an actual intake of 600mg is converted to a requirement-adjusted intake of 300mg: 

\[\frac{600\text{mg}}{1000\text{mg}} \times 500\text{mg}.\]

Table 1 lists a summary of the data for the total sample and for each of the four groups of interest observed at the individual and at the household levels. The individual-level controls that are used in the analysis include household income, household size, an indicator if an individual is currently receiving food stamps, an indicator if an individual is currently lactating or postpartum, and a set of dummies for age, main food preparer’s education level, urban residence, gender, and race. The household-level controls include household income, household size, an indicator for the presence of lactating or postpartum women in the household, an indicator for the presence of an infant, an

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For reference, one cup of liquid milk is set equal to 244 grams.
indicator for the household receiving food stamps, and a set of dummies for the main food preparer’s education and race.

Table 2: Variables and Sample Mean Values

<table>
<thead>
<tr>
<th>Variable</th>
<th>Sample</th>
<th>WIC</th>
<th>Non-WIC</th>
<th>WIC</th>
<th>Non-WIC</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of indiv</td>
<td>2372</td>
<td>526</td>
<td>488</td>
<td>712</td>
<td>646</td>
</tr>
<tr>
<td>Milk/100</td>
<td>3.17</td>
<td>4.68</td>
<td>1.49</td>
<td>4.42</td>
<td>1.82</td>
</tr>
<tr>
<td>Income/$1000</td>
<td>17.83</td>
<td>15.37</td>
<td>16.63</td>
<td>18.68</td>
<td>19.81</td>
</tr>
<tr>
<td>Household size</td>
<td>4.96</td>
<td>4.84</td>
<td>5.45</td>
<td>4.62</td>
<td>5.07</td>
</tr>
<tr>
<td>Food stamp indiv.</td>
<td>0.38</td>
<td>0.56</td>
<td>0.52</td>
<td>0.28</td>
<td>0.24</td>
</tr>
<tr>
<td>PregLactPost indiv.</td>
<td>0.03</td>
<td>0.07</td>
<td>0.00</td>
<td>0.05</td>
<td>0.00</td>
</tr>
<tr>
<td>Age</td>
<td>12.61</td>
<td>3.31</td>
<td>23.63</td>
<td>3.42</td>
<td>22.01</td>
</tr>
<tr>
<td>College</td>
<td>0.30</td>
<td>0.27</td>
<td>0.26</td>
<td>0.32</td>
<td>0.32</td>
</tr>
<tr>
<td>Urban</td>
<td>0.76</td>
<td>0.76</td>
<td>0.75</td>
<td>0.77</td>
<td>0.76</td>
</tr>
<tr>
<td>White</td>
<td>0.47</td>
<td>0.40</td>
<td>0.35</td>
<td>0.56</td>
<td>0.49</td>
</tr>
<tr>
<td>Male</td>
<td>0.49</td>
<td>0.48</td>
<td>0.53</td>
<td>0.48</td>
<td>0.49</td>
</tr>
<tr>
<td>Number of hhlds</td>
<td>1036</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Income/$1000</td>
<td>17.19</td>
<td>15.07</td>
<td>15.21</td>
<td>18.69</td>
<td>18.72</td>
</tr>
<tr>
<td>Household size</td>
<td>4.56</td>
<td>4.64</td>
<td>4.85</td>
<td>4.48</td>
<td>4.71</td>
</tr>
<tr>
<td>Food stamp present</td>
<td>0.44</td>
<td>0.60</td>
<td>0.54</td>
<td>0.33</td>
<td>0.34</td>
</tr>
<tr>
<td>College</td>
<td>0.30</td>
<td>0.27</td>
<td>0.29</td>
<td>0.33</td>
<td>0.34</td>
</tr>
<tr>
<td>Children ages 1-5</td>
<td>0.94</td>
<td>0.98</td>
<td>0.76</td>
<td>0.99</td>
<td>0.98</td>
</tr>
<tr>
<td>Urban</td>
<td>0.76</td>
<td>0.75</td>
<td>0.73</td>
<td>0.78</td>
<td>0.78</td>
</tr>
<tr>
<td>White</td>
<td>0.60</td>
<td>0.54</td>
<td>0.51</td>
<td>0.64</td>
<td>0.61</td>
</tr>
<tr>
<td>Infant present</td>
<td>0.20</td>
<td>0.25</td>
<td>0.42</td>
<td>0.08</td>
<td>0.08</td>
</tr>
<tr>
<td>PregLactPost present</td>
<td>0.15</td>
<td>0.20</td>
<td>0.16</td>
<td>0.12</td>
<td>0.11</td>
</tr>
<tr>
<td>Self-report income</td>
<td>0.16</td>
<td>0.16</td>
<td>0.17</td>
<td>0.16</td>
<td>0.15</td>
</tr>
<tr>
<td>Savings less $5,000</td>
<td>0.94</td>
<td>0.98</td>
<td>0.95</td>
<td>0.91</td>
<td>0.91</td>
</tr>
</tbody>
</table>

In order to deal with the potential endogeneity of WIC program participation in our model, an instrument is required. This instrument must affect the household’s WIC participation decision but not be correlated with unobservables in the consumption equation. Our choice of instrument in this regard is to exploit state-level institutional characteristics of the WIC program in which the individuals reside. Specifically, we make use of information regarding whether or not the state WIC program allows participants to self-declare their income in order to prove eligibility. Less strict states (i.e., those that allow individuals to self-declare) should generally be associated with higher participation rates in WIC. However, allowing households to self-declare income in order to establish WIC eligibility should play no structural role on calcium consumption, conditioned on WIC participation.\(^8\) We also make use of

\(^8\)Owing to confidentiality concerns, our data do not provide state identifiers but does provide region identifiers. To this end, we obtain an average of state policies within each region, using the fraction of WIC participants in state \(s\) within region \(r\) to weight the policy associated with state \(s\). This instrument is not ideal, but should still provide some overall degree of conditional correlation with WIC participation to aid identification. Empirically, we find that this is the case.
a second instrument, which is an indicator denoting if household savings are less than $5000. Our argument in this regard is that families with considerable savings may be less likely to participate in WIC, while levels of asset accumulation should have little to do with calcium consumption, conditioned on current income, WIC participation, education and other demographic controls.

4 Empirical Results

Using the algorithm of section 2 we fit our model, running the Gibbs sampler for 100,000 iterations and discarding the first 10,000 as the burn-in period. The prior hyperparameters used in the calculations are \( \mu_\gamma = 0 \), \( V_\gamma = 100 \), \( \mu_{uv} = 0 \), \( V_{uv} = 100 \), \( a_\epsilon = 3 \), \( b_\epsilon = 1/(2 \times .3) \), \( a_\eta = 3 \) and \( b_\eta = 1/(2 \times .3) \). Generated data experiments were also performed with large sample sizes to suggest that our code is error-free and that our algorithm can adequately recover parameters of the data generating process in these cases. Parameter posterior means, standard deviations and probabilities of being positive associated with the model in (5) - (9) are reported in Table 3.

With respect to WIC participation, the results shown in Table 3 are generally consistent with our prior expectations. Larger households with smaller incomes and infants present in the house are clearly more likely to participate in WIC. Similarly, our instruments appear to play strong roles in the WIC participation decision and operate in the direction that we expect a priori. That is, families living in regions where self-reports of income is likely to provide sufficient proof of WIC eligibility are associated with higher probabilities of WIC participation. Similarly, families with relatively small amounts of savings are also associated with higher probabilities of WIC participation.

We also conduct a variant of the standard “overidentification” test to investigate an aspect of the instrument’s validity. That is, conditioned on the assumption that self-reports of income is a valid instrument, our savings indicator is superfluous in the sense that it is not required for identification. To this end, we then re-estimate the model and include this variable in the latent calcium consumption equation. Doing this, we find a posterior mean (and posterior standard deviation) associated with the Savings < 5000 coefficient equal to -0.15 (.32), and a associated posterior probability of being positive equal to .32. Thus, we do not see a strong role for our asset accumulation variable in the calcium consumption equation, and moreover, under the employed priors, Bayes factors via the Savage-Dickey density ratio do not support its retention in the model. This provides some suggestive evidence, though certainly not formal proof, of the validity of our identification strategy.

With respect to calcium consumption, few variables emerge as strong predictors. Families in larger households tend to consume more calcium through milk while households with higher incomes tend to consume less calcium through milk. Of course, the most important of the coefficients in Table 3
are the coefficients associated with the group identifiers $G_2 \rightarrow G_4$.\(^9\) These findings first suggest, quite sensibly, that non-target members living in WIC households ($G_2$) have a lower (adjusted) calcium intake through milk than target members of WIC households ($G_1$). Surprisingly, however, the results also suggest that non-WIC members, both target and non-target, receive more calcium intake through milk than their WIC counterparts.

Although these results might seem startling, and potentially suggest that the WIC program is ineffec-

\(^9\)Given that $G_1$ (target individuals participating in WIC) represents the excluded category, the coefficients on $G_2 \rightarrow G_4$ should be interpreted relative to this base group.
tive, this is not necessarily the correct interpretation. Individuals participating in WIC may, in fact, use the benefits provided by the WIC program to purchase other products and receive an adequate level of calcium intake through the consumption of these alternate products. What the results do tell us, however, is that the WIC program does not appear to be effective at increasing calcium consumption through milk. In short, the coefficients associated with the group identifiers do not necessarily call into question the effectiveness of the WIC program, but at the same time, and unlike past studies in the literature, they can not be used to speak to its virtues. At a minimum, we find that the presence of the WIC program leads to repackaging of consumption bundles and a substitution away from milk consumption toward other possible foods providing calcium. To our knowledge, these results represent a new contribution to the existing literature on this topic.

Table 3 also shows significant evidence of skew through positive values associated with the skewness parameter $\psi$. The table also shows, quite interestingly, a large, positive value associated with the correlation parameter $\rho$. This suggests, consistent with our prior views, that unobservable factors making a family more likely to participate in WIC also lead that family to consume higher levels of calcium through milk.

| Group | $E(\cdot|y)$ | Std$(\cdot|y)$ |
|-------|-------------|--------------|
| $G_1$ | 470         | 338          |
| $G_2$ | 192         | 338          |
| $G_3$ | 387         | 325          |
| $G_4$ | 183         | 325          |

Table 4: Posterior Predictive Statistics Associated with Calcium Intake for Four Groups

Figure 1: Predictive Posterior Intake Distributions for Four Groups.

Table 4 presents posterior predictive means and standard deviations associated with calcium consumption levels, as described in section 2.3, while Figure 1 plots the entire posterior predictive calcium

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distributions for each of the four groups. When performing these calculations, we set the continuous covariates at sample mean values specific to the “target” or “non-target” populations. (Setting age, for example, to the overall mean of 12.6 would seem inconsistent with both the target and non-target populations, leading us to set the covariates to group-specific means for this exercise). For the binary indicators, we round the target/non-target-specific sample means to either zero or one.

Since these posterior predictive densities account for both the “structural” impacts of WIC participation as well as the role of unobserved confounding, we would expect these predictives to match, to a reasonable degree, the means found in the raw data. A comparison of the entries of Tables 2 and 4 shows that this is (approximately) the case - target members of WIC households and target members of non-WIC households have the highest levels of calcium consumption with posterior means equal to 470 and 387 milligrams, respectively. Similarly, non-target WIC and non-WIC members have lower levels of adjusted calcium intake with posterior means equal to 192 and 183, respectively, which is also broadly consistent with the mean intake levels found in the raw data. Figure 1 and Table 4 also offer little evidence in favor of the potential “leakage” or “spillover” benefits associated with the WIC program; the posterior predictives for the non-target WIC (G2) and non-target non-WIC (G4) individuals are very similar and nearly indistinguishable in Figure 1. Finally, the posterior standard deviations of Table 4 and plots in Figure 1 also reveal considerable heterogeneity associated with the calcium intakes for each of these four groups, with target individuals associated with the highest levels of uncertainty.

While inspection of just the “structural” WIC coefficients on Table 3 would appear to suggest that target non-WIC children will have more calcium consumption through milk than target WIC children, the posterior predictives tell a different story. Like the raw data, these posterior predictives reveal that target WIC children will have the highest levels of calcium consumption through milk. What is responsible for this finding is the role of unobserved correlation - those families that select into WIC possess unobserved factors that also strongly correlate with calcium consumption. This finding is broadly consistent with the idea that the families participating in WIC, holding all else constant, also take great care in the nutritional intakes of their children, and thus would likely consume relatively high levels of calcium even in the absence of WIC. What we have offered in this paper, which to our knowledge is new to this literature, is a model that seeks to separate the influences of unobservables and direct “structural” impacts. When combining these influences, we generate predictions that are consistent with the raw data and the findings of past work on this topic. When separating them, we produce no direct evidence that WIC itself is responsible for increases in calcium consumption and improved overall nutrition. Again, we must interpret this finding with care, as it is certainly possible that the WIC program leads individuals to substitute away from traditional consumption bundles and meet necessary nutritional requirements through other foods. If true, this result does not seem to have been documented in the literature, and has important implications for designing efficient mechanisms for achieving desired nutrient intake levels.
5 Conclusion

In this paper we have described a Bayesian posterior simulator for fitting a two equation treatment-response model and employed this method to investigate the effectiveness of a widely-used food assistance program. This program, commonly denoted as WIC, seeks to improve the nutrition of at-risk low income children and pregnant/breastfeeding mothers. We evaluate this program by focusing on calcium intake through milk consumption and comparing such intake levels across WIC and non-WIC households and individuals. Though this metric is, admittedly, rather narrow, we also recognize that adequate calcium intake is one of the primary focuses of WIC program, and milk is a primary vehicle through which calcium is consumed.

Overall, we find little direct evidence that speaks to the efficacy of WIC. Instead, most of the benefits that might potentially be attributed to the program seem to arise from differences in unobservables across WIC and non-WIC families. Furthermore, we find little evidence associated with possible “spillover” or “leakage” benefits that have been suggested in the literature, as non-target members of WIC households have consumption patterns that are quite consistent with non-target members of non-WIC households. We must interpret our results with caution, however, as it remains possible that WIC benefits lead individuals to substitute away from milk and toward other goods which also provide adequate nutrition. To our knowledge, no studies in the area have attempted to separate the effects of unobservables and direct impacts, yet doing so has clearly been quite important in the context of our application.
References


