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Pricing an Option on Revenue from an Innovation: An Application to Movie Box Office Revenue

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We develop a model for valuing revenue streams from innovations. The stochastic properties of revenue from innovations create a more difficult environment in which to value options than when the underlying is a security. There is no initial revenue, and cumulative revenue cannot decrease. Revenues from innovations are characterized by different lives and different rates of the resolution of uncertainty. A common deterministic model for predicting revenue from an innovation is known as the Bass model. We embed the Bass model in a gamma process, resulting in a stochastic process with moments proportional to the mean of the Bass model. To illustrate this model we choose the valuation of options on movie box office revenue. These options enable film distributors to manage the risk of a movie, and they offer diversification opportunities for investors. We develop the econometric methodology for ex ante parameter estimation and a Bayesian updating scheme using Markov chain Monte Carlo simulation as data after release become available. Call prices obtained using the maximum likelihood (ML) parameter estimates from the full data set closely approximate the average discounted value of ex post call payouts that would have occurred at option maturity.

Key words: innovation; Bass model; option pricing; gamma process; risk management; movie revenues; movie box office receipts; nondecreasing process; Bayesian update
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1. Introduction

Financial instruments play a vital role in a market economy. They allow investors to separate and trade sources of risk in an efficient manner. Moreover, beyond securities and conventional derivatives are other instruments that enhance risk allocation and help complete the market. In this paper, we develop and demonstrate the technology for an instrument that permits the trading of risk associated with the revenue from an innovation, as opposed to the risk of a price or rate derived from a financial asset.

Direct claims on revenue streams present an alternative to conventional claims. A review of the literature shows that these instruments already trade, and there is much interest in further development of this market. To date, most of the instruments have taken the form of securitized claims on a revenue stream, typically offered through special purpose vehicles.[1] The practitioner literature reveals a growing demand for securitization of products such as drug royalties, franchising revenue, and intellectual property and options on the latter.[2]

Securitization creates a market for trading the underlying, and derivative contracts can help complete the market. The entertainment industry has been an innovator in creating instruments such as Bowie bonds that were first offered in 1997 and in which payments are contingent on a stream of revenues from album sales. Bowie bonds have also been offered by at least seven other entertainers. In 1992 Walt Disney Company placed a Eurobond issue with a coupon containing an option on the revenue from a portfolio of movies. Of course, bonds with embedded options contingent on a revenue stream have existed for many years and are widely used in municipal finance. Although these instruments are bonds, they contain options, so pricing the bonds requires pricing these options. In recent years, there has been discussion in the risk industry of creating options on the revenue

[1] See LeClair and Schulman (2006) for details of how these revenue-based instruments are created.

[2] See Dorris (2003), Hillery (2004), and Edwards (2001) for discussions of securitizing these revenue streams, and Kahn (2000) for options on intellectual property.

stream from movies that would trade apart from any fixed-income offering (see Conway 1997, Patel 2004). Owners of oil fields and gold mines would also seem to be natural users of options based on revenue, provided the options can be priced.

Whether these options trade separately or embedded in bonds, pricing options when the underlying is a revenue stream poses some major challenges. Cumulative revenue streams are nondecreasing in time. Thus, models for revenue differ from standard models, in which asset prices or interest rates increase or decrease. Specifically, note that the drift term in an Itô diffusion is $O(dt)$, whereas the volatility term is $O(\sqrt{dt})$. Locally, the diffusion term dominates so that approximately one-half of process changes will be negative. Although a variety of models for nondecreasing processes exist, two likely candidates are those based on Poisson arrivals of nonnegative jumps and those based on the gamma process.

We specifically consider nondecreasing and short-lived processes that are usually investigated under the rubric of the "diffusion of innovations." The pioneering work for the adoption of an innovation is due to Bass (1969), who proposed a three-parameter deterministic model based on the maximum possible number of adopters, a coefficient of innovation, and a coefficient of imitation. We embed the standard Bass model as the mean in the gamma process. The result is a stochastic process whose moments are proportional to the mean of the Bass model. Derivative prices follow in a straightforward manner for innovations with returns uncorrelated with market returns.

We apply the gamma model with Bass embedded parameters to movie box office revenue. As we discuss in §5, movies are an excellent setting in which to study options on revenue. Movies behave like short-lived innovations. They typically realize the majority of their box office revenue in the first 20 to 25 weeks after initial release. We also find that movie returns are uncorrelated with market returns, and thus we discount expected payoffs under the physical measure using the risk-free rate. Initial estimates of the process parameters are obtained from historical box office data using a sample of 244 movies from 1998 to 2000. Using in-sample parameter estimates and all 244 movies, we find that call prices obtained using maximum likelihood (ML) parameter estimates from the full data set closely approximate the average discounted value of ex post call payouts that would have occurred at option maturity.

For our out-of-sample estimations, we employ a Bayesian updating algorithm using Markov chain Monte Carlo (MCMC) to adjust the initial estimates as revenue observations arrive. We apply the prerelease estimation, the updating, and the resulting option pricing formula to three movies released in 2001 and show how the option price profiles converge under updating to the discounted terminal payoff.

The objectives of this paper are (1) to develop a stochastic process and option pricing model for revenue consistent with the diffusion of innovations literature, (2) to provide tractable econometrics for estimating and updating model parameters using movie box office revenue as our example, (3) to evaluate the model fit, and (4) to demonstrate how the model would be applied.

This paper is organized as follows: §2 introduces a deterministic model for revenue that forms the basis for the stochastic model developed in §3; §4 develops option pricing formulae; §5 addresses movie box office revenue processes; §6 develops the econometric methodology; in §7, we price options on example movies and explore their characteristics, as well as the fit of our model; and §8 concludes.

2. A Deterministic Model for the Adoption of an Innovation

We define the adoption of an innovation up to time t as the number of people who have bought the product at time t. To fix notation, we denote the adoption of an innovation in the deterministic model introduced in this section by $n(t)$. In the stochastic model introduced in the next section, we use $N(t)$ for the adoption. We assume a constant average unit price a for each adoption. Then the revenue up to time t is given by $R(t) = an(t)$ in the deterministic model and by $R(t) = aN(t)$ in the stochastic model.

In this study, we use the Bass (1969) model as the deterministic baseline model for the adoption process. Bass assumes that there are two forces influencing the adoption of an innovation. One force is independent of the previous number of adopters, and the other force is positively influenced by the previous number of adopters. Consider first an individual adopter. The probability of an event (adoption) in the interval $[t, t+dt]$, given that the event has not occurred previously, is given by $h(t)dt$, where $h(t)$ is the hazard function. The Bass hazard model is

$$h(t) = \frac{f(t)}{1-F(t)} = p + qF(t), \tag{1}$$

where $f(t)$ is the density function of the time of adoption and $F(t)$ is the cumulative density up to time t, i.e., $f(t) = F'(t)$ with initial condition $F(0) = 0$. The structure of the model is determined by p and q. The parameter q must be nonnegative and p must be positive. Both parameters must be finite if the density function is to be nondegenerate.

The first force in the adoption process, p, has been called the coefficient of innovation. It is the decision to adopt independent of the actions of others. Bass calls the second force, q, the coefficient of imitation. Lekvall and Wahlbin (1973) have referred to these coefficients, respectively, as the external and internal influence in the adoption process. The solution of the ordinary differential equation (1) is

$$F(t) = \frac{1 - e^{-(p+q)t}}{1 + (q/p)e^{-(p+q)t}}. \tag{2}$$

The density function for the time of adoption is then

$$f(t) = \frac{dF(t)}{dt} = \frac{(p+q)^2 p e^{-(p+q)t}}{(p + qe^{-(p+q)t})^2}, \quad 0 < t < \infty.$$

If the potential population, m, is fixed, the total number of adoptions up to time t under the deterministic model is $n(t) = mF(t)$ and the rate of adoptions is $mf(t)$. The same solution can be obtained by defining $F(t) = n(t)/m$ as the fraction of individuals who have adopted by time t. From (1) the differential equation for $n(t)$ is given by

$$\frac{dn(t)}{dt} = (m - n(t))\Big(p + q\frac{n(t)}{m}\Big), \tag{3}$$

$$n(t) = m\Big(\frac{1 - e^{-(p+q)t}}{1 + (q/p)e^{-(p+q)t}}\Big) = mF(t).$$

From Equation (3) it is clear that the rate of adoptions is proportional to the remaining nonadopters. The coefficient of proportionality is $p + qn(t)/m$.

3. A Stochastic Revenue Model for Innovations

We embed the adoption $n(t)$ from the Bass model of §2 in a gamma process $N(t)$ such that the first moment of the process is the number of adopters according to the Bass model:

$$\mathbb{E}N(t) = n(t).$$

Therefore, $N(t)$ does not satisfy the differential equation (3), but $\mathbb{E}N(t)$ does. The stochasticity of the process captures random deviations from the expected adoption. The increments $N(t) - N(s)$, $t > s$, are gamma distributed with mean equal to $m(F(t) - F(s))$ and variance $m(F(t) - F(s))\beta$, where β is a volatility parameter. The following paragraphs outline the construction of the process.

Consider the probability space $(\mathbb{R}_+, \mathcal{F}_t, \mathbb{P})$, on which there is defined a gamma process $N(t)$ in continuous time. That is, $N(t)$ has independent increments and the probability density of $\Delta N := N(t+h) - N(t)$ is given by

$$f_{\Delta N}(x) = \frac{x^{\alpha-1}e^{-x/b}}{b^{\alpha}\Gamma(\alpha)}, \quad 0 < x < \infty,$$

where $\alpha = \mu^2 h/\nu$, $b = \nu/\mu$, $\mu, \nu \in \mathbb{R}$ and $\Gamma(\alpha) = \int_0^\infty t^{\alpha-1}e^{-t}\,dt$ is the gamma function. Then, $\mathbb{E}(\Delta N) = \mu h$, $\mathrm{Var}(\Delta N) = \nu h$, and we can write symbolically for the limit $h \to 0$:

$$\mathbb{E}dN = \mu dt,$$

$$\mathrm{Var}(dN) = \nu dt.$$

The density $f_N(\cdot)$ gives rise to the probability measure $\mathbb{P}$, given by

$$\mathbb{P}(N(t) \in dx) = \frac{x^{t\alpha-1}e^{-x/b}}{b^{\alpha t}\Gamma(t\alpha)}dx.$$

The filtration $\mathcal{F}_t$ is given by the sigma fields generated by $N(t)$, $\mathcal{F}_t := \sigma(N(s), s \in [0, t])$. The process $N(t)$ is a special case of a Lévy process (Bertoin 1996, p. 73; Tsilevich et al. 2001).

To capture the Bass model in the first moment, we define μ and ν such that for $s < t$ they satisfy

$$\mathbb{E}\Delta N = \mu(t - s) := m(F(t) - F(s)),$$

$$\mathrm{Var}\Delta N = \nu(t - s) := \beta m(F(t) - F(s)),$$

where $F(t)$ is the Bass model given by Equation (2). Therefore, as $s \to t$, $\mu dt = mdF(t)$ and $\nu dt = \beta m dF(t)$ so that our model can be expressed as

$$dN(t) \sim Gamma\Big(\frac{mdF(t)}{\beta}, \beta\Big). \tag{4}$$

For arbitrary increments of length $t - s > 0$,

$$N(t) - N(s) \sim Gamma(\alpha(s, t), \beta), \tag{5}$$

where $\alpha(s, t) := m(F(t) - F(s))/\beta$. Consistent with the rapid resolution of uncertainty in an innovation, the variance $\beta m(F(t) - F(s))$ likewise diminishes with $t - s$.

Skewness and kurtosis of the increments are given by

$$\mathrm{Skew}(N(t) - N(s)) = \frac{2}{\sqrt{\alpha(s, t)}}, \tag{6}$$

and

$$\mathrm{Kurtosis}(N(t) - N(s)) = 3 + \frac{6}{\alpha(s, t)}. \tag{7}$$

Furthermore, under independent increments and appropriate choice of parameters, gamma variates reproduce under addition. Unlike geometric Brownian motion, zero initial values do not cause problems.[3]

[3] Many studies incorporate the gamma process into the volatility term. See, for example, Carr et al. (2005), Heston (1993), Madan et al. (1998), Madan and Milne (1991), Madan and Seneta (1990). Other applications of the gamma process in financial modeling can be found in Todorov and Tauchen (2006) and Shaliastovich and Tauchen (2006).

4. Option Value for Innovation Revenue

Consider European options on revenue from innovations that are cash-settled at expiration. Minimum values and put-call parity can be established in the common fashion by creating combinations of options, the underlying revenue stream, and risk-free bonds. An option on a cumulative revenue stream can be irreversibly in-the-money, meaning that at a given time during its life, cumulative receipts can already exceed the exercise price, guaranteeing that the option will expire in-the-money. It is straightforward to show that American call options may be exercised early. American put options will be exercised immediately if they are in-the-money. If cumulative revenue exceeds the strike, American puts are irreversibly out-of-the-money and are worthless. In this paper, we price only European options.

We assume a continuous-time capital asset pricing model world with constant investment opportunity set and with revenue uncorrelated with the market (Merton 1972).[4] Thus, the risk is diversifiable and not priced. Under this setup, the call option pricing formula at time s for an option expiring at time T can be obtained directly under the physical measure as

$$\begin{aligned} C(s,T) &= ae^{-r(T-s)}\mathbb{E}[\max\{0, N(s)+U(s,T)-K_N\} \mid \mathcal{F}_s] \\ &= ae^{-r(T-s)}\int_d^\infty (N(s)+u(s,T)-K_N) f_U(u)\,du, \end{aligned} \tag{8}$$

where a is the constant unit price, r is the risk-free rate, $N(s)$ is adoption through time s, $U(s,T) := N(T)-N(s) \sim \mathit{Gamma}(\alpha(s,T),\beta)$ is remaining adoption until time T, K_N is strike price in adoption units, and $d=\max\{0, K_N - N(s)\}$. Using the analytic expression for the gamma density f_U, the call value can be written as

$$C(s,T) = ae^{-r(T-s)}(\alpha\beta\Pi_1 + (N(s)-K_N)\Pi_2), \tag{9}$$

where $\alpha\beta = m(F(T)-F(s))$ and

$$\alpha\beta\Pi_1 = \int_d^\infty u f_U(u)\,du = \alpha\beta\left(1-\frac{\Gamma_{d/\beta}(\alpha+1)}{\Gamma(\alpha+1)}\right).$$

Here $\Gamma_d(\alpha) = \int_0^d t^{\alpha-1}e^{-t}\,dt$ is the incomplete gamma function. Also note that the cumulative gamma distribution function is $F_U(d) = \Gamma_{d/\beta}(\alpha)/\Gamma(\alpha)$. The probability of finishing in-the-money is

$$\Pi_2 = \int_d^\infty f_U(u)\,du = 1-\frac{\Gamma_{d/\beta}(\alpha)}{\Gamma(\alpha)}.$$

The call formula can also be cast in terms of forward prices. Because revenue is assumed to be independent of market returns and interest rates, the forward price of remaining revenue at time s is $J(s,T) := a\mathbb{E}(U(s,T)\mid\mathcal{F}_s) = a(n(T)-n(s)) = am(F(T)-F(s))$, so that

$$C(s,T) = e^{-r(T-s)}(J(s,T)\Pi_1 - (K-R(s))\Pi_2), \tag{10}$$

where $K := aK_N$ and $R(s) = aN(s)$.[5] This characterization is similar to that of Bakshi and Madan (2002) in their model for catastrophe options. Note that the forward price is on remaining revenue, and the effective exercise price is $K-R(s)$.

If $R(s) > K$, the option is irreversibly in-the-money, $d=0$, and so the option price becomes

$$C(s,T) = e^{-r(T-s)}(J(s,T)+R(s)-K).$$

Note that volatility does not matter in this case.

Boundary conditions are satisfied at $s=T$. At option expiration, $\alpha(T,T)=0$ and $\Pi_2 = 1-\Gamma_{d/\beta}(0)/\Gamma(0) = 1-\int_0^{d/\beta}(e^{-t}/t)\,dt/\int_0^\infty(e^{-t}/t)\,dt$, as well as $J(T,T)=0$. If the option is in-the-money, $d/\beta=0$, $\int_0^\infty(e^{-t}/t)\,dt \to \infty$, $\Pi_2=1$; and by (10), $C(T,T) = R(T)-K$. If the option is out-of-the-money, $d>0$. Rewrite the equation for Π_2 as

$$\Pi_2 = \frac{\int_{d/\beta}^\infty (e^{-t}/t)\,dt}{\int_0^\infty (e^{-t}/t)\,dt}.$$

However, $\int_{d/\beta}^\infty (e^{-t}/t)\,dt$ is finite for $d/\beta > 0$, so $\Pi_2 = 0$, and by (10), $C(T,T)=0$.

The value of a European put is given by put-call parity or computed directly as

$$\begin{aligned} P(s,T) &= ae^{-r(T-s)}\int_0^d (K_N - N(s) - u(s,T)) f_U(u)\,du \\ &= e^{-r(T-s)}\left((K-R(s))\left(\frac{\Gamma_{d/\beta}(\alpha)}{\Gamma(\alpha)}\right) - J(s,T)\left(\frac{\Gamma_{d/\beta}(\alpha+1)}{\Gamma(\alpha+1)}\right)\right) \\ &= e^{-r(T-s)}((K-R(s))(1-\Pi_2) - J(s,T)(1-\Pi_1)). \end{aligned}$$

5. Movie Box Office Revenue

There is growing interest in instruments in which the underlying is a revenue stream. We explore the pricing of options on revenue streams using movies as a case in point. Movies behave like short-lived innovations. The uncertainty about box office revenue resolves rapidly after release, often after the first few weekends. Box office revenue is a function of the price of a ticket and the number of people who attend a movie. The ticket price should be fairly stable over the short life of the theater run of a movie, and the

[4] In §6.1, we verify this assumption for our data set.

[5] The value of the revenue stream at time s is $V(s,T) = e^{-r(T-s)}(R(s)+J(s,T))$.

average ticket price across all theaters can be easily determined. This allows us to focus on movie attendance, and hence base our approach on a model that captures the adoption of a movie, by which we mean the number of people who pay to watch the movie in theaters.

Most movies also have a second life, generating cash flows through television rights, video rentals and sales, and merchandise. Another major source of revenue is foreign box office receipts, video sales, and rentals, which can occur almost simultaneously with domestic cash flows. Our focus is exclusively on U.S. box office revenue, even though they represent less than 20% of total revenue (Ravid and Basuroy 2004). The stochastic process and the option pricing model developed in §§3 and 4 are in principle adaptable to options on other sources of revenue. Options on total revenue from all sources, however, will be more difficult to price due to the different timing of the releases of the different products. The time-series characteristics of data from other products will likely differ from those of box office revenue.

5.1. Movie Box Office Revenue and Risk

Movies provide interesting opportunities in both the buying and selling of the risk that arises from their revenue streams. On the buy side, there is a large and growing hedge fund and private equity industry that makes substantial investments in movies (see Kelly 2006). Most of the structures are securitized revenue streams. One major factor that motivates such demand is the potential that investments in movies provide for improving the risk-return trade-off of existing positions by adding exposure in an area with low or nearly zero correlation with more conventional markets. As we show in §6.1, this point is upheld in our data.

If revenue is uncorrelated with the market, then so are the corresponding options, and thus demand for derivatives is consistent with demand for revenue. In a world with continuous trading, options would be redundant because they can be created by a dynamically adjusted combination of the securitized investment and a risk-free bond. Continuous trading is a mathematical construct, however, and in any case the market for securitized investments in movies is still relatively thin in comparison to more conventional and established markets. Hence, there is no way for these private equity and hedge fund investors to establish speculative positions with limited losses and unlimited gains or to benefit from poor performance of the underlying. Calls and puts, respectively, permit the establishment of these types of payoff profiles.

Besides the private equity and hedge fund industry that already invests in movies, there is a substantial subculture of followers of movie revenue statistics, and tremendous interest in speculating on these figures, as evidenced by the 1.6 million participants and 42,000 daily trades on the Hollywood Stock Exchange, which offers virtual stock, options, and futures on the revenue of movie stars and films (King 2006). Although virtual investing is far removed from actual investing, the data generated by the Hollywood Stock Exchange are used by studios to gauge market sentiment about movies, stars, directors, etc., indicating interest in a more efficient way to process market information.

However, buyside demand is only part of the picture. Sellside interest can come from either the desire to earn a profit by meeting buyside demand or from a need for reducing the risk that a studio or distributor assumes when it makes movies. Movies are primarily owned by large corporations. In spite of the fact that their shareholders would presumably hold broadly diversified portfolios, large movie corporations have shown considerable interest in hedging the risk of movie revenue. There is certainly much theory and evidence, such as Graham and Rogers (2002) and Froot et al. (1993), to justify why corporations hedge. Specifically, hedging reduces the costs of financial distress, bankruptcy, and informational asymmetry, supports value-creating investment activity, and improves performance evaluation. Tufano (1996) finds that some corporate hedging exists because of managerial risk aversion.

Movie studios typically separate the production and artistic decisions from the financing and distribution decisions, which parallels the financial theory of separation of investment from financing decisions.[6] Studios manage risk in a variety of ways. One decision variable is which films to produce. Ravid and Basuroy (2004) suggest that studio executives produce more R-rated movies than G-rated movies, in spite of the greater profitability of the latter (Ravid 1999). This decision has a diversification aspect, but the literature has noted that corporate diversification provides only limited risk reduction when cash flows are correlated. Insurance has also been used but insurance contracts are essentially the same as options, but with added layers of regulation and greater contract complexity. Moreover, as Phillips (2004) notes, some insurance contracts designed to lay off the risk of U.S. movies to European investors resulted in disastrous bankruptcies in recent years. Studios also lay off some of the risk through their arrangements with theaters (Filson 2005).

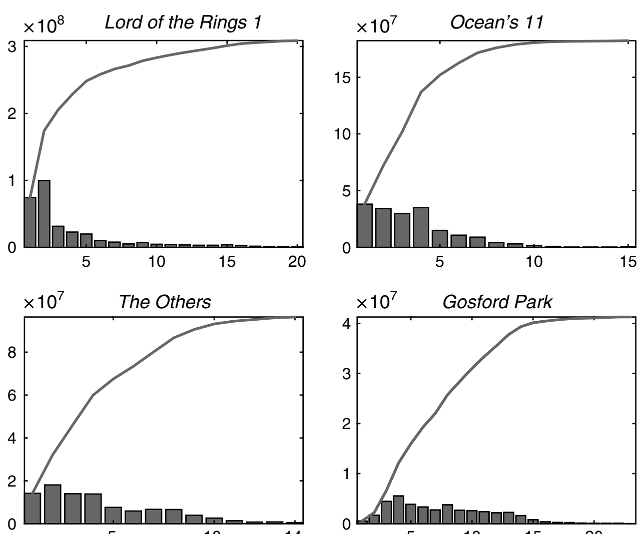
[6] Our focus in this paper is on the risk associated with movie distribution and not on any costs, risks, or benefits of financing. Fee (2002) examines the financing of a movie, likewise observing a type of separation between the artistic decisions and the source of funding.

Derivative instruments broaden the base of participants who will accept the risk and therefore make the market for movie risk transfer more efficient. As Merton (1998) notes, movies are an excellent example of a real option, which suggests that this industry is a natural field for applying option pricing theory beyond its more conventional uses. Movie box office revenue cannot, however, be modeled like stock returns and interest rates. In the following section, we identify the empirical characteristics of movie revenue.

5.2. Empirical Characteristics of Movie Revenue

Movies often have an exponentially declining revenue process. In so-called platform releases where the number of initial screens is small, however, the revenue process has a hump shape (Sawhney and Eliashberg 1996). Figure 1 shows weekly U.S. box office revenue (bars) and cumulative revenue (lines) for four example movies. *The Lord of the Rings*: *The Fellowship of the Ring* and *Ocean's 11* are two of the most successful movies released in 2001 and examples of highly advertised wide releases. They had marketing budgets of \$40 million and \$30 million and were shown on 3,359 and 3,075 initial screens, respectively; the run times in theaters were 36 weeks and 21 weeks. The graphs show only the first 20 and 15 weeks, respectively, because these weeks show by far the largest part of the dynamics. Afterward, the movies had reached a saturation point. In both cases, the number of theaters decreased after the opening weekend. On the other hand, *Gosford Park*, also released in 2001, is

Figure 1 Weekly and Cumulative U.S. Box Office Revenue for Four Example Movies



Notes. Shown are the weekly incremental (bars) and cumulative (lines) box office revenues from four example movies released in 2001. The data are obtained from http://www.the-numbers.com and http://www.imdb.com.

an example of a platform release. It had a low budget, \$18 million in total, and was initially released on only nine screens. Over the following weeks, the distributor increased the number of screens to 131, 518, 658, ..., with a maximum of 918 in the 11th week. The movie lasted 23 weeks in theaters. *The Others* falls in between: released on 1,678 screens, it reached a maximum of 2,843 theaters in Week 6 and was shown for 14 weeks. Figure 1 illustrates that these characteristics have a direct influence on the shape of the revenue process. Wide releases have exponential dynamics in the weekly revenue and approach their maximum adoption faster than platform releases, which reach a maximum in weekly revenue during the first few weeks and then decline slowly.

Conditional models for total movie receipts have been the subject of a number of papers in the economics and marketing literature. The predominant structure of these papers is to forecast revenue as a function of the characteristics of the movie and certain time-series properties. The most recent work includes the models of Walls (2005), Elberse and Eliashberg (2003), Simonoff and Sparrow (2000), Ravid (1999), and Sawhney and Eliashberg (1996).

Another stream of the literature examines the unconditional cross-sectional distribution of total movie revenue. DeVany and Walls (1999, 2002) find that the stable Paretian distribution with characteristic exponent less than two is a good fit for most movie genres, suggesting that the variance of total revenue is infinite. Compared to the Gaussian model, total revenue is thus characterized by fat tails or extreme events. Similarly, DeVany and Walls (2004) find that the stable Paretian model for movie *profit* is better than the Gaussian model either with or without the presence of a "star."

Although there is supporting evidence for the stable Paretian model, it poses challenges whenever a finite second moment is needed. In addition, there are alternative explanations for fat tails in the literature of financial economics. For example, Bollerslev (1986, 1987) documents that time-dependent variance can generate fat-tail behavior in returns. In this paper, we pursue the route of time-dependent variance rather than extreme value distributions.

Stochastic processes that are positive and nondecreasing have been developed to model Asian options and catastrophe options. The underlyings for these options are the sum of positive prices and positive index numbers, respectively.

The literature on Asian options is extensive. Curran (1994) develops an approximation by conditioning on the geometric mean, whereas Milevsky and Posner (1998) derive the density of an infinite sum of correlated log-normals and use this result to approximate the finite sum density. A number of papers develop general pricing results for Asian options using Laplace transforms or characteristic functions. These papers offer new insights and especially useful results for pricing Asian options on interest rate processes. Papers of this genre include those of Geman and Yor (1993), Ju (1997), and Bakshi and Madan (2000).

The process for cumulative box office receipts is similar in some respects to the process for catastrophe indices. Bakshi and Madan (2002) use a characteristic function approach to develop an option pricing model for a catastrophe index that consists of a positive deterministic mean component and nonnegative Poisson regulated jumps. Although the receipts process has similar features, it differs in that there is quick resolution of uncertainty. Therefore, variance disappears with time. Further, weekly receipts time series are very short. These issues present difficult econometric problems for a Poisson process with time-dependent jump intensity and time-dependent deterministic component (e.g., the Bass model). In simulations, we found that the gamma process developed in §3 has much better econometric properties. The next section outlines our estimation approach.

6. Parameter Estimation

Operationalizing the model is a nontrivial step. The gamma model requires the initial estimation and the update of the parameter vector

$$\theta := (m, p, q, \beta).$$

These parameters have an intuitive interpretation as market potential, attenuation of the revenue process, acceleration of the revenue process due to imitation, and the volatility parameter, respectively. Their estimation, however, is not as straightforward as, for example, the estimation of constant volatility in the Black-Scholes model.

Our solution to this problem proceeds as follows: (1) estimate the parameters by maximum likelihood using a data base of movies (§6.2); (2) regress the estimated parameters on movie characteristics such as budget, initial screens, and rating (§6.3); (3) use the movie characteristics and regression parameters to estimate the initial parameter vector for a set of out-of-sample movies; and (4) use a Bayesian framework to update the parameter vectors for the example movies as real-time data becomes available (§6.4).[7]

[7] Details of the estimation and results not reported here are available in the online appendix, provided in the e-companion. An electronic companion of this paper is available as part of the online version that can be found at http://mansci.journal.informs.org/.

6.1. Data

Our data set consists of the 100 most successful movies for each of the years 1998, 1999, and 2000 in terms of U.S. box office revenue. The data are obtained from http://www.the-numbers.com and the Internet Movie Database (http://www.imdb.com). We require the sample period to cover the entire life span of the movie. That is, we do not use movies released prior to 1998 but still playing in 1998, and we also do not use movies still playing in early 2001. This requirement leaves us with a sample of 244 movies. This procedure introduces a selection bias toward successful movies into the estimates. The sample is representative of wide releases with large marketing costs, but not of the overall movie population that contains many platform releases. Thus, although the sample size may seem small for the entire population of movies, at over 200 degrees of freedom it is sufficiently large to reliably estimate the parameters for wide releases. A time series of annual average movie ticket prices was obtained from the website of the National Association of Theater Owners, http://www.natoonline.org.

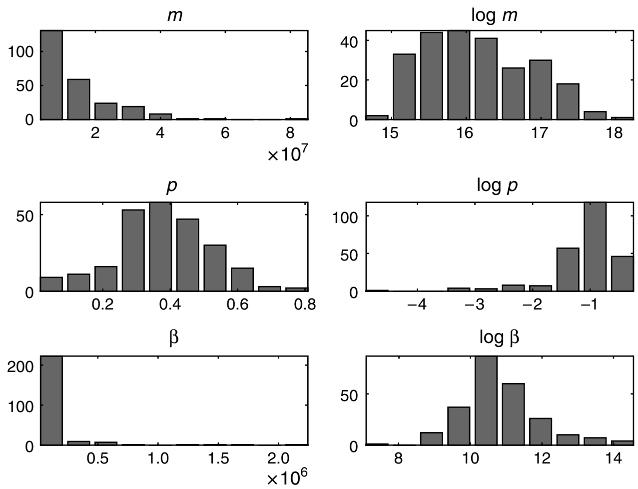
Using our data set, we constructed an index of weekly movie returns during 1998–2000. Comparing this index to the weekly returns from the S&P 500, we found no significant correlation in bivariate Granger-causality tests. This finding confirms the assumption stated in §4. The details of the test are available in the online appendix.

6.2. Maximum Likelihood Estimation

Maximum likelihood estimation of the revenue model in Equation (5) is straightforward. Given a time series of box office revenue $R(t)$, $t = 1, \ldots, T^*$, we divide the series by the average ticket price a to obtain the adoption series $N(t)$, $t = 1, \ldots, T^*$. We use weekly data, so estimates of parameters p and β are thus in units per week.

For some movies, the literature has documented that there is an imitation effect in adoption, which translates to $q \neq 0$ in the Bass model (Eliashberg and Shugan 1997, Basuroy et al. 2003). This imitation effect is particularly pronounced in platform releases that rely more on word of mouth than on large marketing efforts. In our study, we focus on wide releases with larger marketing budgets in which the imitation effect is not prevalent, so we assume $q = 0$ in the Bass model. The reason for this procedure is that there are well-documented econometric problems with the fully specified Bass model. Boswijk and Franses (2005) found that if one adds a Brownian motion error to a deterministic Bass mean function, one cannot estimate q consistently if there are no data beyond the maximum rate of adoption. Even though our model has a different stochastic structure, we expect a similar

Figure 2 Histograms of the Estimated Parameters and Their Logarithms for 244 Movies



Notes. Shown are histograms of maximum likelihood estimates of m, p, and β (and logs thereof) given by Equation (11). Data for the 244 movies are obtained from http://www.the-numbers.com and http://www.imdb.com and are described in §6.1. The parameters estimated are the maximum number of admissions m, the coefficient p of innovation from the Bass model, and the volatility parameter β from the gamma process that captures the unexpected innovations.

result to hold, which will adversely affect the update procedure, where only the first few observations are available.

In the case of movie revenue, we also have to deal with substantial small-sample effects. In simulations, we found that for the small sample size of the movie box office data problem, the quality of the ML estimators of the process parameters is very poor if we let $q \neq 0$, whereas we obtain reasonable accuracy if we require $q = 0$. For larger sample sizes, the maximum likelihood estimation (MLE) is well behaved. These simulations are available in the online appendix.

The likelihood function for the parameter vector $\theta = (m, p, \beta)$ is given by

$$L(\theta \mid \{N(t)\}_{t=1,\ldots,T^*}) = \prod_{t=1}^{T^*} \frac{1}{\beta^{\alpha(t-1,t)}\Gamma(\alpha(t-1,t))} u(t-1,t)^{\alpha(t-1,t)-1} e^{u(t)/\beta}, \quad (11)$$

where $\alpha(t-1,t) = m(F(t) - F(t-1))/\beta$ and $u(t-1,t) = N(t) - N(t-1)$, $N(0) = 0$. We estimate the parameter vector by maximum likelihood for the 244 movies in our sample. Figure 2 shows the histograms of the maximum likelihood estimates and their logs for the movies in our data base.

6.3. Estimation Prior to Any Revenue Data

If there are no revenue, and thereby adoption, data available, the adoption process must be forecast using historical data from previously released movies. Variables with forecast power discussed in the context of movies are, to name a few, genre, rating, country of release, budget, presence or absence of stars, number of screens, sexual or violent content, and sequel (Jones and Ritz 1991, Sawhney and Eliashberg 1996, Neelamegham and Chintagunta 1999).

Here we use a regression model where the logs of the maximum-likelihood estimates of the parameters of the movies in a database are regressed on the number of initial screens, the log of the budget, and on dummies for the ratings PG, PG-13, and R. There are no unrated movies in our sample; hence, the baseline case is G-rated. We keep the numbers of regressors to a minimum to reduce collinearity (Hill and Adkins 2001). The estimation equations are, thus,

$$\log\hat{\theta}_i = \log\begin{bmatrix}\hat{m}_i\\ \hat{p}_i\\ \hat{\beta}_i\end{bmatrix} = \begin{bmatrix}\xi_i b^m + \eta_i^m\\ \xi_i b^p + \eta_i^p\\ \xi_i b^\beta + \eta_i^\beta\end{bmatrix}, \quad i = 1, \ldots, N, \quad (12)$$

where the $(1 \times K)$ regressor vector ξ_i consists of K explanatory variables, $b^{(\cdot)}$ are parameter vectors, N is the number of movies in the database, and $\eta_i^{(\cdot)}$ is white noise with mean zero and variance σ_i^2. We assume that there is no covariance across parameters and across movies. Plugging in the characteristics ξ of the movie for which the initial adoption is to be estimated yields a point estimate $\hat{\theta}_0$ for θ at time $t = 0$ before revenue data are available.

Table 1 reports the results from the regression of the maximum likelihood parameter estimates on the movie characteristics.

Several papers have found a significant influence of G or PG ratings on movie returns (Ravid 1999, Simonoff and Sparrow 2000, DeVany and Walls 2002, Fee 2002); our results are in accordance with these findings.

Table 1 Estimation of Model Parameters from Movie Characteristics

	log(*m*)		log(*p*)		log(β)	
Constant	10.56***	(0.71)	0.01	(0.54)	4.97***	(1.12)
Initial screens	1e−4**	(6e−5)	4e−4***	(4e−5)	−5e−4***	(1e−4)
log(Budget)	0.32***	(0.04)	−0.14***	(0.03)	0.45***	(0.07)
Rating						
PG	−0.21	(0.20)	0.12	(0.15)	−0.74**	(0.32)
PG-13	−0.16	(0.18)	0.33***	(0.14)	−1.08***	(0.29)
R	−0.26*	(0.18)	0.43***	(0.14)	−1.11***	(0.28)
R^2	0.29		0.36		0.23	

Notes. Shown are regression results for Equation (12) for the 244 sample movies over the period 1998–2000. Data are obtained from, http://www.the-numbers.com and http://www.imdb.com. The full set of movies contains the 100 most successful movies released in each of the years 1998, 1999, and 2000 that completed their theater runs during that same time period. Elimination of movies with incomplete data reduced the sample to 244 movies. *, **, and *** denote significance at the two-sided 99%, 95%, and 90% levels, respectively.

We also considered dummy variables for genre, graphic content, and sequels, and found the choice of regressors used in Table 1 the best in terms of avoiding collinearity, minimizing the Bayes information criterion, and individual variable significance. It is not our intention, however, to be authoritative about the choice of regressors or even about the linear regression specification. Instead, our objective is to demonstrate a method to obtain initial estimates that can be updated in a Bayesian framework and used for option valuation. Practitioners in the field should have access to more information and might consider alternative specifications.

6.4. Update Scheme for Incoming Revenue Observations

The uncertainty in the box office revenue stream resolves rapidly during the first weeks after the release. To revalue the option, it is necessary to update the initial estimates $\hat{\theta}_0$ as the revenue observations come in. Because the process is non-Gaussian, a nonlinear Kalman filter approach is not feasible. Therefore, we employ a full Bayesian update approach and evaluate the involved integrals by MCMC simulation.

We use only the initial number of screens as a regressor, and not the weekly time series of screens. Therefore, we do not introduce an endogeneity bias into the estimates, at the cost of not using available data. The weekly number of screens is a decision variable that is agreed upon by distributor and theater owners. The division of box office revenue between distributor and theater owners usually follows a formula that depends on the number of screens and the time the movie has been shown. We do not address the consequences of these dynamics in this paper. Suppose the distributor wants to use the breakeven point as strike price for the option. For example, if the budget of the movie was \$$B$ and over the lifetime of the movie a fraction g of box office revenue will accrue with the distributor, the option's strike price could be set at \$$B/g$ (or any multiple of that number, depending on the fraction of revenue that is to be hedged). We ignore that g is a function of screens and time. This issue could pose a problem if the option matures within the first few weeks after release when the number of screens has substantial influence on revenue dynamics. If the maturity is set well after the fast resolution of uncertainty, this assumption should be innocuous. A study of the influence of the number of screens is an interesting topic for further research.

To set up the Bayesian update, we explore the statistical properties of the estimated parameters from the database movies, as shown in Figure 2. We find that a lognormal distribution best describes the empirical distributions of the estimates of m and β. For p, on the other hand, a normal distribution seems appropriate. Table 2 reports the estimated parameters of the prior distributions from a fit of the 244 estimated parameter vectors.

Table 2 Estimated Parameters of the Prior Distributions

	$\hat{m}$	$\hat{p}$	$\hat{\beta}$
	lognormal	normal	lognormal
μ	16.1514	0.3788	10.8144
σ	0.7095	0.1436	1.0828

Notes. Means and standard deviations of the estimated parameters for each of the 244 sample movies over the period 1998–2000. These are the estimates of the maximum number of admissions m, the coefficient p of innovation from the Bass model, and the volatility parameter β from the gamma process. The parameters m and β are assumed to be lognormally distributed and p is assumed to be normally distributed.

Both the normal and lognormal distributions are determined by a location parameter μ and a scale parameter σ. The interpretation of the location parameter μ as the expected value of the log of the parameter θ_i allows us to plug in the initial estimate $\hat{\theta}_{0,i}$ obtained from the regression model and then update the initial estimate by convoluting with the likelihood equation (11) as revenue observations arrive. For example, the likelihood for the first adoption observation $N(1)$, obtained by dividing the first revenue observation $R(1)$ by the average ticket price a, is given by

$$f(N(1)-N(0)\mid\theta)=\frac{1}{\beta^{\alpha(0,1)}\Gamma(\alpha(1))}u(0,1)^{\alpha(1)-1}e^{u(1)/\beta}, \tag{13}$$

where $\alpha(0,1)=m(F(1)-F(0))/\beta$, $u(0,1)=N(1)-N(0)$, and $N(0)=0$. The likelihood for the first two adoption observations is given by

$$\begin{aligned}&f(N(2)-N(1), N(1)-N(0)\mid\theta)\\&\quad=f(N(2)-N(1)\mid\theta)f(N(1)-N(0)\mid\theta),\end{aligned} \tag{14}$$

and so forth.

In the Bayesian update scheme, the marginal posterior distribution of the updated parameter vector given the first adoption observation is obtained by integrating over the product of the prior distribution and the likelihood equation (13). For the second observation, we consider the product of the prior distribution with Equation (14), and so forth. The expected value of the marginal posterior distribution then serves as the point estimate of the updated parameter vector. For example, for the adoption parameter m, the marginal posterior density of the update given the first adoption observation is given by

$$\begin{aligned}&f(m\mid N(1)-N(0),\hat{\theta}_0)\\&\quad=\int_p\int_\beta f(N(1)-N(0)\mid\theta)f(\theta)\,dp\,d\beta,\end{aligned} \tag{15}$$

where $f(\theta)$ is the full (multivariate) prior distribution. Evaluating this integral by numerical integration is infeasible because we have estimates of the marginal prior distributions only. In particular, the covariance of the parameters, here p and β, is unknown. Therefore, we employ MCMC simulation (Liu 2001 and Gilks et al. 1996) to evaluate Equation (15) and its counterparts for p and β.

7. Applications and Characteristics of Revenue Options

In this section, we first explore how well our proposed revenue process (4) fits the data from our sample of movies. Then we apply the option pricing method to the three movies *The Fellowship of the Ring*, *Ocean's 11*, and *The Others* of Figure 1. These movies were all released in 2001, the year after our sample period ends. We do not apply the method to *Gosford Park*, because this movie is a platform release.

7.1. Data Fit

To evaluate the model fit, we use real revenue data from the 244 movies in our sample. We compute the discounted payoffs $e^{-rT}\times\max\{0, R_i(T)-K_{i,j}\}$ for an array of call options with strikes set at multiples of the movie budget. Because the budgets are different for all movies, we standardize the budget to \$100 million by multiplying the revenue by 100 million/budget$_i$.[8] We set the grid of strikes to $K_j = x_j$ 100 million. The x_j cover an equidistant grid between 0.2 and 7. These multiples span most of the movies in our sample. Then we average the discounted payoffs over the 244 movies and plot the average against the different strike multiples x_j (see Figure 3).

We compare the discounted payoffs to the option prices (9) implied by our model using the maximum likelihood estimated parameters. If the model (4), the option price formula (9), *and* the maximum likelihood estimator are appropriate, the two resulting lines should be close to each other. Figure 3 shows that the two lines are visually indistinguishable, indicating a good fit of our model. The difference between the two lines is two orders of magnitude smaller, i.e., less than 1%.

7.2. Application of the Update

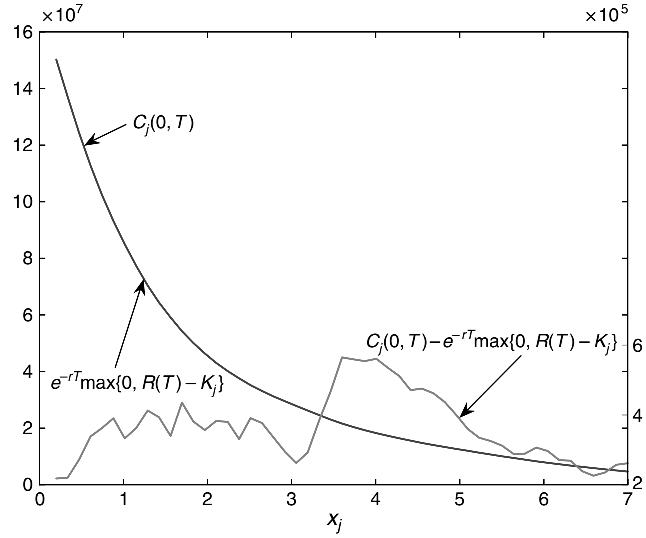
We study the three movies *The Fellowship of the Ring*, *Ocean's 11*, and *The Others* from Figure 1 as prototypes. Table 3 shows the updating procedure. Using the characteristics of the movies in the regression equation, we obtain the initial estimates $\hat{\theta}_0$ of the parameter vectors reported in the first row. We update the estimate as the observations of the weekly revenue data come in by evaluating (15) by MCMC. The priors are chosen according to Table 2, where the means are set to the initial estimate $\hat{\theta}_0$. The initial values of the Markov chain are also set to $\hat{\theta}_0$. Note how the update picks up underestimations or overestimations in particular of m and β very quickly and corrects them in the right direction in the first few weeks. Also note how much of the uncertainty resolves after about five to six weeks for all three movies: the parameter estimates become stable and begin to move toward the maximum likelihood estimate given all data reported in the last row of the tables.

This procedure is fairly flexible and allows the use of other priors. For example, for *Ocean's 12*, released in 2004, one could use the maximum likelihood parameters of *Ocean's 11* as priors instead of the regression results. Similar adaptations could be made for the two sequels of *The Lord of the Rings*.

7.3. An Example

In this section, we illustrate how the model would be used to obtain an initial option value. Consider

Figure 3 Average Model Prices vs. Average Discounted Payoffs



Notes. The plots are cross-sectional averages of 244 movies. The curve denoted by $e^{-rT}\max\{0, R(T)-K_j\}$ is the average of realized call payoffs. The curve $C_j(0,T)$ is the average of the call option prices given by the model and Equation (9). $C_j(0,T)$ overlays $e^{-rT}\max\{0, R(T)-K_j\}$ in the plot. The curve denoted by $C_j(0,T)-e^{-rT}\max\{0, R(T)-K_j\}$ is the averaged error and plotted against the scale on the vertical axis on the right. In computing the averages, we introduce the cross-sectional index $i = 1, \ldots, 244$. The strike prices $K_{i,j} = x_j\text{Budget}_i$ are set at multiples x_j of the movie budget; the $x_j = 0.2 + 0.136j$, $j = 0, \ldots, 50$, are plotted on the x-axis. Most of the movies in our sample earned between 0.2 and 7 times their budget. The curves are then computed as the averages of $C_{i,j}(0,T)$, $e^{-rT}\max\{0, R_i(T)-K_{i,j}\}$, and $C_{i,j}(0,T)-e^{-rT}\max\{0, R_i(T)-K_{i,j}\}$.

[8] Note that the option price formula is homogeneous under scalar multiplication. The option price is given by $e^{-rT}\mathbb{E}\max\{0, R-K\}$. The function $\max\{0, R-K\}$ is homogeneous, $\mathbb{E}$ is a linear operator, and e^{-rT} is a constant. Therefore, the option price is homogeneous in R.

Table 3 Update of the Parameter Estimates for *The Fellowship of the Ring*, *Ocean's 11*, and *The Others*

	The Fellowship of the Ring			*Ocean's 11*			*The Others*		
	$\widehat{m}$	$\hat{p}$	$\hat{\beta}$	$\widehat{m}$	$\hat{p}$	$\hat{\beta}$	$\widehat{m}$	$\hat{p}$	$\hat{\beta}$
$\hat{\theta}_0$	29,398,914	0.3825	42,607	27,980,841	0.3444	48,932	10,890,824	0.2480	46,681
Week 1	41,328,908	0.7775	77,225	26,178,633	0.5675	74,837	10,151,541	0.5450	82,870
Week 2	44,391,295	0.5816	1,461,957	35,072,749	0.3608	139,600	12,931,686	0.4466	249,946
Week 3	48,545,490	0.5210	1,208,977	43,304,843	0.2617	125,208	15,063,577	0.3557	260,639
Week 4	52,087,207	0.4652	985,290	46,384,682	0.2461	332,450	17,188,784	0.2920	279,351
Week 5	55,362,629	0.4156	846,817	45,549,959	0.2437	296,471	18,719,201	0.2546	213,189
Week 6	56,720,731	0.4015	737,636	44,571,319	0.2586	279,200	19,668,193	0.2349	165,739
Week 7	58,078,833	0.3903	647,096	44,542,535	0.2472	250,417	21,004,801	0.2117	140,344
Week 8	58,957,605	0.3832	580,522	43,117,750	0.2634	263,369	22,448,338	0.1901	124,305
Week 9	59,623,341	0.3627	567,207	42,311,811	0.2750	264,809	22,735,709	0.1863	107,597
Week 10	60,289,078	0.3498	535,252	41,764,923	0.2859	269,126	22,488,437	0.1895	102,251
Week 11	60,715,149	0.3362	516,611	41,347,562	0.2962	273,444	21,880,280	0.1994	110,270
Week 12	61,300,997	0.3241	497,971	40,901,417	0.3069	290,714	21,218,659	0.2112	125,641
Week 13	61,407,515	0.3116	495,308	40,771,891	0.3125	287,835	21,118,413	0.2145	120,295
Week 14	61,540,662	0.2986	497,971	40,743,107	0.3150	279,200	20,911,239	0.2203	121,631
Week 15	61,487,403	0.2836	516,611	40,786,283	0.3138	263,369			
Week 16	61,540,662	0.2712	521,937						
Week 17	61,673,809	0.2623	513,948						
Week 18	61,700,439	0.2543	508,623						
Week 19	61,647,180	0.2467	505,960						
Week 20	61,806,956	0.2409	492,645						
MLE	62,259,657	0.2425	537,915	36,796,883	0.3144	267,687	20,222,885	0.2200	108,265

Notes. The updates provide estimates of the model parameters that incorporate the arrival of weekly revenue data. The estimates are obtained by MCMC with priors based on the results in Table 2 and means set to the initial estimates. These are the estimates of the maximum number of admissions m, the coefficient p of innovation from the Bass model, and the volatility parameter β from the gamma process. MLE is the estimator using all of the data.

The Others as an example. If the distributor Miramax wishes to reduce some of the risk, two reasonable strategies are to sell a call (referred to as covered call) or buy a put (referred to as protective put). We consider a covered call that is sold the instant before the movie is released. Let the option expire in six weeks. At the time of release, the continuously compounded risk-free rate was about 3.6%, based on the U.S. Treasury bill secondary market. Plugging the characteristics (1,678 initial screens, production budget $17 million plus $10 million in advertising) into the regression equation (12), we obtain the initial estimates $\widehat{m}_0 = 10{,}890{,}824$, $\hat{p}_0 = 0.2480$, and $\hat{\beta}_0 = 46{,}681$.

The average ticket price in 2001 is given as $a =$ $5.65. Hence, the forecast $a\widehat{m}_0$ of total revenue is about $61.5 million. Given the production budget and the $10 million spent on advertising and distribution, the movie is forecast to be profitable by U.S. box office receipts alone, and indeed it was, with a total revenue take of about $96 million.

An exercise price set equal to the movie budget gives an option that would finish deep in-the-money should expected revenue be realized. Such an option would likely not be of much interest to the buy side. Consequently, we set the exercise price at the expected revenue according to the initial parameter estimate: $amF(6) = \$5.65 \times 10{,}890{,}824 \times 0.7742 = \$47{,}638{,}969$ or 8,431,676 in units of adopters. Inserting these parameters into the option-pricing formula gives a call option value of $1,406,888.

Hence, a distributor who sold options on the entire position would collect revenue of $1,406,888 upfront. At expiration, six weeks later, total revenue was $73,422,887. Thus, the option would have expired with a payoff

$$\max\{0; \$73{,}422{,}887 - \$47{,}638{,}969\} = \$25{,}783{,}918.$$

After the remaining eight weeks of the movie, the distributor would then be left with net revenue of $96,471,845 − $25,783,918 = $70,687,927. Given production and distribution costs of $27,000,000 and adding the option premium, Miramax would have realized a profit of about $45 million. Of course, it might not sell options on the entire movie proceeds. In addition, some of the proceeds go to the theaters according to a sharing formula.

If the holder of the option wished to sell the option during the six-weeks lifetime, the updated parameters from Table 3 would be used to value the option. The call was irreversibly in-the-money from Week 4 on, with approximately $60 million in cumulative revenue in Week 4. Table 4 presents the sequence of call option prices according to the realizations of cumulative revenue and the parameter updates.

Table 4 Update of the Call Option Price for ***The Others***

Week	Option price	Updated	$\hat{\theta}_0$
0	$C(0,6)$	1,406,888	1,406,888
1	$C(1,6)$	623,300	1,492,742
2	$C(2,6)$	9,462,206	8,083,123
3	$C(3,6)$	17,677,290	13,824,536
4	$C(4,6)$	25,641,653	21,211,206
5	$C(5,6)$	26,515,019	29,674,504
Payoff	$C(6,6)$	25,783,918	25,783,918

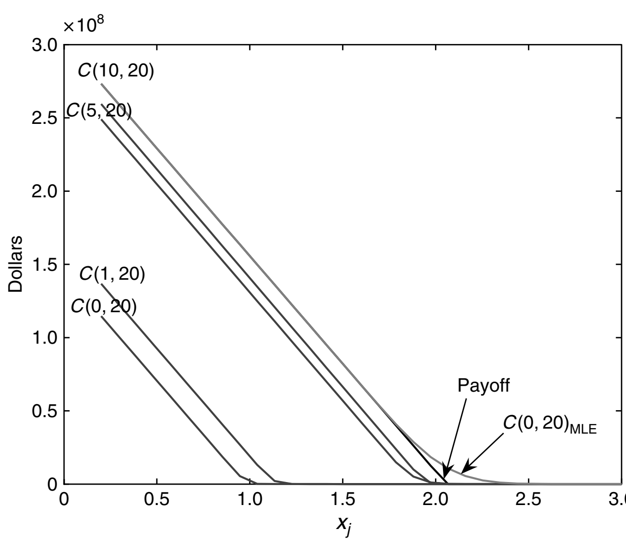
Notes. Call option prices for *The Others* during the six weeks of the life of the option. The values in column 3 reflect updated parameter estimates, whereas the values in column 4 use the initial parameter estimates of $m = 10{,}890{,}824$, $p = 0.2480$, and $\beta = 46{,}681$. The updated estimates use MCMC to incorporate the arrival of new information. Both sets of estimates use the realized revenue. The risk-free rate is held constant at 3.6%, and the option exercise price is the expected revenue of $amF(6) = \$5.65 \times 10{,}890{,}824 \times 0.7742 = \$47{,}638{,}969$.

7.4. Option Price Update for the Prototype Movies

We repeat this computational exercise for each of the three movies and calculate the call option prices for a grid of strikes similar to Figure 3. The expiries are set at Week 20 for *The Fellowship of the Ring*, Week 15 for *Ocean's 11*, and Week 14 for *The Others*. The grid of strikes is set at multiples x_j of the budget, as in Figure 3. We then plot the option prices $C(t, T)$ for selected t against the strike multiples x_j.

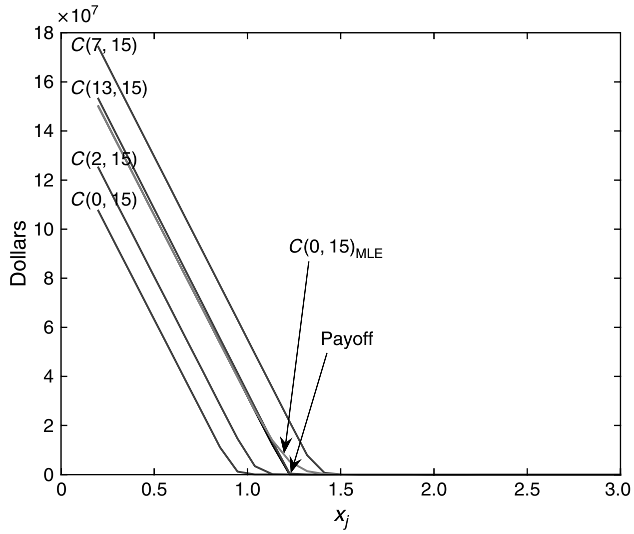
Figures 4–6 show the graphs. The option price profiles all approach the payoff with elapsed time. For *The Lord of the Rings*, the approach is strictly from below, but for the other two movies the line switches to the top of the payoff profile. As a benchmark, each graph also shows the option price $C(0, T)_{\text{MLE}}$ computed from the maximum likelihood estimator. This estimator uses all revenue data, which are not available at time $t = 0$.

Figure 4 Option Price Update for ***The Fellowship of the Ring***



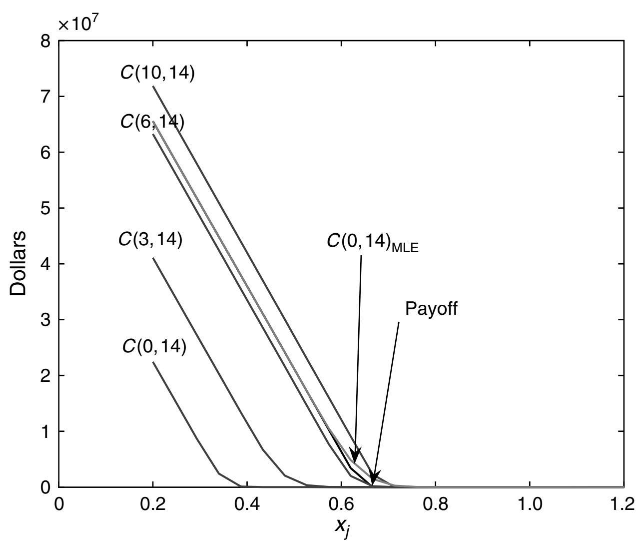
Notes. Selected option price profiles are shown. $C(0, 20)$ is computed from the initial estimate $\hat{\theta}_0$, the $C(t, 20)$ are computed from the cumulative revenue at t and the updated parameter estimates from Equation (15). $C(0, 20)_{\text{MLE}}$ gives the MLE estimate of the call at time $t = 0$ (using data that are not available at $t = 0$). Strike prices are $K_j = x_j$Budget.

Figure 5 Option Price Update for ***Ocean's 11***



Notes. Selected option price profiles are shown. $C(0, 15)$ is computed from the initial estimate $\hat{\theta}_0$, the $C(t, 15)$ are computed from the cumulative revenue at t and the updated parameter estimates from Equation (15). $C(0, 15)_{\text{MLE}}$ gives the MLE estimate of the call at time $t = 0$ (using data that are not available at $t = 0$). Strike prices are $K_j = x_j$Budget.

Figure 6 Option Price Update for ***The Others***



Notes. Selected option price profiles are shown. $C(0, 14)$ is computed from the initial estimate $\hat{\theta}_0$, the $C(t, 14)$ are computed from the cumulative revenue at t and the updated parameter estimates from Equation (15). $C(0, 14)_{\text{MLE}}$ gives the MLE estimate of the call at time $t = 0$ (using data that are not available at $t = 0$). Strike prices are $K_j = x_j$Budget.

8. Summary and Conclusions

In this paper, we develop a model for valuing revenue streams from product innovations. The stochastic properties of revenue from innovations create a different environment in which to value options than when the underlying is a security. There is no revenue figure at the start, and cumulative revenue cannot decrease. Revenue from innovations is characterized by pronounced nonstationarity.

The standard deterministic model for predicting revenue from an innovation is due to Bass (1969). We embed the Bass model as the mean function in the gamma process, giving a stochastic process with moments proportional to the mean of the Bass model and a free parameter to specify process variance. We assume and verify empirically that returns from revenue innovations are uncorrelated with market returns, and thus obtain an option pricing model.

To illustrate this model we examine the valuation of options on movie box office revenue. Primary among the problems is the fact that revenue data are unavailable before movie release. We develop the econometric methodology for ex ante parameter estimates using regressions on movies' characteristics, such as budget, initial screens, and ratings. After release, we compute Bayesian updates using MCMC simulation. Call prices obtained using ML parameter estimates from the full data set closely approximate the average discounted value of ex post call payouts that would have occurred at option maturity.

The gamma process with an embedded time-varying mean function could prove useful in valuing derivatives on other random variables with nonnegative increments such as credit derivatives and catastrophe options. The credit option underlying would be cumulative losses on a debt portfolio, whereas the catastrophe option underlying is a nondecreasing catastrophe index.

Other applications of the gamma process can be inferred from an examination of the practitioner literature. This literature has discussed the securitization of the underlying revenue streams for a broad range of applications such as pharmaceuticals, intellectual property, and entertainment revenue. Potential applications could also include revenue from an oil field, a gold mine, or any resource or activity that generates a nondecreasing stream of cash. Of course, when these revenue streams are already allocated in the form of standard equity claims, investors can trade securities that presumably reflect the characteristics of these revenue streams. Corporations are large and complex entities, however, whose cash flows are usually a mixture of sources of revenue. As LeClair and Schulman (2006) argue, there is considerable potential and benefit for corporations to sell off portions of their revenue streams to interested investors. Where claims on revenue streams exist, the options help complete the market by providing an efficient means of risk transfer.

9. Electronic Companion

An electronic companion to this paper is available as part of the online version that can be found at http://mansci.journal.informs.org/.

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