Mean Reversion Expectations and the 1987 Stock Market Crash: An Empirical Investigation

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Abstract

After the stock market crash of 1987, Fischer Black proposed an equilibrium model in which he explained the crash by inconsistencies in the formation of expectations of mean reversion in stock returns. This study derives testable hypotheses implied by this explanation. An Ornstein-Uhlenbeck process that is consistent with Black's model is estimated on daily stock index data before and after the crash of 1987. The results strongly support the hypothesis. Simulations show that on Friday Oct 16, 1987, a crash of 20 percent or more had a probability of more than seven percent.

JEL classification: G10, C22

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1 Introduction

The report of the Brady Commission emphasized the role of portfolio insurance strategies in the stock market crash of 1987 (Brady et al. 1988). According to the Brady Report, dynamic hedging strategies were triggered on a large scale on Black Monday and led to a downward cascade. This interpretation was criticized because only a small fraction of the market volume was managed using dynamic hedging strategies and the elasticity of stock demand implied by the magnitude of the crash seemed unreasonable (Leland 1988, Rubinstein 1988, Brennan and Schwartz 1989).

A small number of studies interpreted portfolio insurance differently (Grossman 1988, Black 1988, Gennotte and Leland 1990, Jacklin, Kleidon, and Pfleiderer 1992). According to the theory developed

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in this literature, the drop in the level of the stock market *prior* to the crash resulted in adjustments of the dynamic hedges that revealed the volume of portfolios under insurance. This share was much larger than the average investor expected. Therefore, during the bull market before the crash, more stock market transactions were made to mirror a put option and not because of fundamental information. This was a piece of fundamental information in itself. The crash is explained by the exposure of an incorrect estimation of the share of portfolios under dynamic hedging strategies. Such an incorrect estimation is possible because portfolio insurance is unobservable. The transactions in stocks (that is, usually index futures) and bonds do not reveal that their point is to synthesize an option.

Black (1988) connected this idea to the concept of mean reversion in stock returns. Unlike mean reversion in stock price volatility, mean reversion in stock returns is a controversial subject. The serial correlation in returns necessary to make them revert to a mean return contradicts the hypothesis of efficient markets. Several studies, however, have shown that underperforming stocks outperform the market in later periods, where "later" can mean anything between three and five years. Vice versa, outperforming stocks seem to underperform at the same time horizon. This "contrarian" view implies negative serial correlation in returns (De Bondt and Thaler 1985, De Bondt and Thaler 1987, Fama and French 1988, Poterba and Summers 1988, Jegadeesh 1990, Cutler, Poterba, and Summers 1991, Chopra, Lakonishok, and Ritter 1992, Lakonishok, Shleifer, and Vishny 1994, Pesaran and Timmermann 1995, Pesaran and Timmermann 2000, Richards 1997, Balvers, Wu, and Gilliland 2000, Balvers and Wu 2006). Another strand of literature on "momentum" strategies has found that on shorter horizons between one month and one year, portfolios of outperforming stocks seem to outperform later and underperforming stocks underperform later. This implies positive serial correlation in returns (Lo and MacKinlay 1988, Jegadeesh and Titman 1993, Jegadeesh and Titman 1995, Chan, Jegadeesh, and Lakonishok 1996, Rouwenhorst 1998, Chan, Hameed, and Tong 2000, Lee and Swaminathan 2000, Grundy and Martin 2001, Jegadeesh and Titman 2001, Lewellen 2002). Both types of serial correlation can be consistent with mean reversion. For example, a covariance stationary autoregressive process with complex roots has an oscillating autocorrelation function. Thus, while the process reverts to its mean and this reversion is governed by the (sum of the) autoregressive parameters, it is possible to measure positive serial correlation at certain time horizons and negative serial correlation at other horizons.

Different explanations are presented for mean reversion in returns, most prominently over- and underreaction to news and time varying required returns as a result of consumption smoothing (Black 1990, Cecchetti, Lam, and Mark 1990, Lo and MacKinlay 1990, De Long, Shleifer, Summers, and Waldmann 1990, Barberis, Shleifer, and Vishny 1998, Hong and Stein 1999, Gatev and Ross 2000).

A number of econometric problems have been discussed. Summers (1986) makes the point that available statistical methods have no power to discriminate between the null hypothesis of no mean reversion and a slowly mean reverting alternative. Early studies that find evidence of serial correlation in returns often used variance ratio tests and autoregressions of return series. These methods tend to reject the null hypothesis of no mean reversion too often (Richardson and Stock 1989, Richardson 1993, Kim, Nelson, and Startz 1991, Kim, Nelson, and Startz 1998, Kim and Nelson 1998). Lo and Wang (1995) study option valuation with mean reverting stock returns.

In Black's (1988) model of the stock market crash of 1987, the underestimation of portfolio insurance translates to an underestimation of mean reversion. When the true size of insured portfolios becomes known, two things happen: The mean reversion parameter in the stock price model must be adjusted, and the stock price path must be corrected to where it would have been if the underestimation had not occurred. This correction in the path is the stock market crash.

So far, the implications of this mean reversion explanation of the 1987 crash have not been explored empirically. The theory predicts that after the stock market crash, mean reversion must have been higher than before the crash. Also, before the crash two periods of different mean reversion should be identifiable: one period of relatively higher mean reversion before the underestimation sets in and one of relatively lower mean reversion right before the crash.

The aim of this paper is to provide evidence from stock index data for Black's hypothesis. We derive a testable hypothesis and specify a stock return model that allows for mean reversion and is consistent with Black's and other equilibrium frameworks. The model is estimated on daily S&P 500 index data around the stock market crash of 1987. The results strongly support Black's explanation. For about five years after the crash, mean reversion was significantly higher than before the crash. During the period 1982–1986, which was identified in the Brady Report as the bull market that led up to the crash, a significantly higher mean reversion than during the year 1987 itself is measured. The Brady Report characterized the boom in 1987 as exaggerated. A generalized likelihood ratio test for a parameter change point finds evidence of a change in the mean reversion regime in early 1987, supporting the segmentation of the Brady Report. Simulations of the model for the 1982–1986 and the January 1987–October 1987 periods result in a probability of more than seven percent for a crash of 20 percent or more. A correction of minus 10 percent or more had a probability of over 40 percent. These probability estimates can be obtained without having to assume heavy tailed distributions for returns.

The paper is organized as follows. Section 2 specifies and discusses a mean reverting model for stock returns. Section 3 outlines Black's explanation of the crash and obtains a testable hypothesis. The section also discusses the events of the week prior to the crash of 1987 in the light of the mean reversion explanation. Section 4 reports the results of the estimation of the model around the 1987 crash and the results of the simulation of the model. Section 5 concludes.

2 A Mean Reversion Model for Stock Returns

An intuitive way to think about mean reversion in stock returns is to assume that the returns process reacts to any deviation from its long term mean. If the return is above the mean in one period, there is a force that pushes it down in following periods, if the return is below the mean, it is pushed up. Similar to Metcalf and Hassett (1995), we use a diffusion model that contains a mean reversion term for the purpose of estimation using stock market data.

The mean return induces a certain stock price, denoted by ϑ_t, which can be interpreted as an estimator of the fundamental value of the stock or stock index. It is

$$\vartheta_t = S_0 e^{\mu\, t},$$

where S denotes the stock price. Consider the return process given by

$$\frac{dS_t}{S_t} = \mu dt + \lambda \frac{\vartheta_t - S_t}{S_t} dt + \sigma dW_t. \tag{1}$$

Here, the term $(\vartheta - S)/S$ measures the deviation of the return process from the long term mean μ. The parameter $\lambda \geq 0$ controls the speed with which the return is pushed back to the mean μ. The average time to revert to the mean is $1/\lambda$ units of time. W_t is standard Brownian Motion. It is shown in the Appendix (Lemma 1) that the expected value of the price process satisfying (1) is

$$\mathbb{E} S_t = S_0 e^{\mu\, t} = \vartheta_t.$$

The process satisfying

$$d \log S_t = \tilde{\mu} dt + \lambda(\log \tilde{\vartheta}_t - \log S_t) dt + \sigma dW_t, \tag{2}$$

where $\tilde{\mu} = \mu - \sigma^2/2$ and $\tilde{\vartheta}_t = S_0 e^{\tilde{\mu} t}$ is a first order approximation to (1), as shown in the Appendix (Lemma 2). The solution to model (2),

$$\log S_t = \log S_0 + \tilde{\mu}\, t + \sigma \int_0^t e^{-\lambda(t-u)} dW_u, \tag{3}$$

is an Ornstein-Uhlenbeck process. Hence, (2) is a Vasicek-type model for log prices (Vasicek 1977). The unconditional distribution of the log price process is given by

$$\log S_t \sim \mathcal{N}\left(\tilde{\mu}\, t + \log S_0, \frac{\sigma^2}{2\lambda}\right). \tag{4}$$

The process is non-stationary. The higher the mean reversion λ, the smaller the variance because the process will stay within a narrower corridor around its mean with the same probability. For purposes of estimation, the conditional distribution of the log returns $r_t := \log S_t - \log S_{t-1}$ given information through time $t-1$ is relevant. It can be read directly from the model (2):

$$r_t \sim \mathcal{N}\left(\tilde{\mu} + \lambda(\log \tilde{\vartheta}_{t-1} - \log S_{t-1}), \sigma^2\right). \tag{5}$$

To estimate the model, maximize the log likelihood

$$L(\theta; \{S_t\}_{t=1,\ldots,T}) = -\frac{T}{2}\log\sigma^2 - \frac{1}{2}\sum_{t=2}^{T}(r_t - \tilde{\mu} - \lambda(\log\tilde{\vartheta}_{t-1} - \log S_{t-1}))^2. \tag{6}$$

Here, T denotes the number of observations, $\theta = (\tilde{\mu}, \lambda, \sigma)'$ is the parameter vector, and $\tilde{\vartheta}_t = S_0 e^{\tilde{\mu}\, t}$ is the fundamental value as above. The derivatives of (6) are readily calculated.

The unconditional distribution of the log returns is given by

$$r_t \sim \mathcal{N}\left(\tilde{\mu}, \frac{\sigma^2}{2\lambda}(e^{\lambda} - 1) + \frac{\sigma^2}{2\lambda}e^{-2\lambda(t-1)}(1 - e^{-\lambda})\right),$$

so that for $t \to \infty$, the stationary distribution is obtained as

$$r_t \overset{t\to\infty}{\sim} \mathcal{N}\left(\tilde{\mu}, \frac{\sigma^2}{2\lambda}(e^{\lambda} - 1)\right).$$

The maximum likelihood estimator $\hat{\theta}$ is asymptotically normal and the usual statistical inference of maximum likelihood estimation applies.

The mean return $\tilde{\mu}$ is not trivial to estimate (Merton 1980). Hence, the samples considered must be chosen carefully to make sure that the estimated mean return is relevant to the analysis. We will use a change point detector and segmentations proposed in the Brady Report for this purpose.

One might argue that if $\vartheta_t = S_0 e^{\mu\, t}$ were an estimator of the fundamental value, the asset would not trade far above or below this value. In other words, a non-negligible distance $\log\vartheta_t - \log S_t$ should not occur. The equilibrium models mentioned in the introduction (for example, Black 1989, Cecchetti, Lam, and Mark 1990) develop a theoretical justification. Further, White (1990) observed for the case of the 1929 stock market crash that during the boom that preceded the crash, fundamentals were very difficult to evaluate. This was mainly because many new companies entered the stock market that had virtually no dividend history. A similar case can be made for the internet boom at the turn of the century. The quality of an estimator for the fundamental value that uses any type of historical long term mean is questionable in situations like this. In the following section, we will suggest another explanation why the process may move far away from the mean return for prolonged periods of time as a consequence of adaptive learning of mean reversion expectations.

3 A Mean Reversion Theory of the 1987 Stock Market Crash

The purpose of this section is to derive a testable hypothesis from Black's (1988) explanation of the 1987 stock market crash. We describe a framework of heterogeneous agents with different expectations of mean reversion. A subset of agents observes the market and adapts their a-priori expectations to those they infer from the market strategies of other agents. This adjustment can be understood as adaptive learning (Evans and Honkapohja 2001).

3.1 Mean Reversion Expectations

Let there be two groups of agents characterized by different risk aversion. The more risk tolerant group (and its market share) is denoted α; the more risk averse group is denoted β. When understood as market shares, $\alpha + \beta = 1$, i.e. there are only these two groups. Note that we use the notation α and β twofold: As a symbol for the set of agents but also as the market share. It should be clear from the context which one is meant. The specific form of the utility function of the agents does not matter for the arguments presented here.

Both groups form expectations of the mean reversion parameter λ in process (2). This Ornstein-Uhlenbeck process is the continuous time equivalent of a first order autoregressive process. It can be seen as a reduced form approximation to any autocorrelating process. Thus, the arguments set out in this section can be used to extend any general equilibrium model that features serial correlation in the returns of a risky asset. For example, process (2) used in this study is a version of Black's (1989) equation (2). Similarly, the equilibrium returns in the discrete time framework of Cecchetti, Lam, and Mark (1990) also display first order partial autocorrelation (their equation (13)).

There is an immediate connection between mean reversion expectations and strategies. Consider for example Figure 1 and a situation where investors have budget constraints. After a positive change in returns, an investor who expects slow mean reversion has a relatively larger long position than an investor who expects fast reversion. The latter sells to realize profits from the high price level that she expects to be of only short duration. An investor who expects slow mean reversion has relatively smaller short positions than an investor who expects fast reversion. The latter will keep or even expand short positions to realize high revenue that can be covered soon at lower prices. The investor expecting slow reversion will expect a longer wait before short positions can be covered at lower prices. Intermediate upward moves could even be possible that could force her to cover at higher prices. In summary, after a positive change in returns, investors with expectations of fast reversion exercise more selling pressure than those who expect prices to revert slowly. If the majority expects fast reversion, the selling pressure will decrease the price and implement the fast reversion into the actual price process. The case of a negative change in returns works symmetrically.

We model the expectations of mean reversion of the two groups α and β as discrete random variables with probabilities $p_\alpha(\lambda_i)$ and $p_\beta(\lambda_i)$ and means $\mathbb{E}_\alpha\lambda = \sum_{i\in\alpha}\lambda_i p_\alpha(\lambda_i)$ and $\mathbb{E}_\beta\lambda = \sum_{i\in\beta}\lambda_i p_\beta(\lambda_i)$, respectively. Here $\sum_{i\in\alpha,\beta}$ denotes summation over all agents i in groups α or β, respectively. In this sense, the probabilities $p_\alpha(\lambda_i)$ and $p_\beta(\lambda_i)$ have a frequentist interpretation: Every agent i expects a specific value λ_i of the mean reversion parameter. This induces a distribution of mean reversion expectations $p_\cdot(\lambda_i)$ in group α and in group β.

The equilibrium price process is characterized by the weighted average of the mean reversion expected by the two groups:

$$\begin{aligned}\lambda_a = \mathbb{E}\lambda &= \sum_{i\in\,\alpha\cup\beta}\lambda_i(\mathbb{1}_\alpha p_\alpha(\lambda_i) + \mathbb{1}_\beta p_\beta(\lambda_i)),\\ &= \alpha\mathbb{E}_\alpha\lambda + \beta\mathbb{E}_\beta\lambda,\end{aligned}$$

where λ_a denotes the actual, data generating mean reversion parameter and $\mathbb{1}_{\alpha,\beta}$ is the indicator function that takes value 1 if individual i is in group α or β, respectively. The assumption that the actual process parameter is equal to the mean of the expectations of the agents is common in the adaptive learning literature with heterogeneous agents (for example, Evans, Honkapohja, and Marimon 2001).

We denote the strategy of an agent i that depends on her mean reversion expectation λ_i by $\pi(\lambda_i)$. The market clearing condition is

$$\sum_{i\in\alpha}\pi(\lambda_i) + \sum_{i\in\beta}\pi(\lambda_i) = 0. \tag{7}$$

Note that since within each group, λ is a random variable, the groups may clear separately. That is, the sums on the right hand side of (7) may be equal to zero individually.

We describe four periods of different information: An initial period $[0, t)$, a period $[t, t + h)$ where group β makes a mistake in adapting their mean reversion expectations, a period $[t + h, t_c)$ in which the effect of the mistake builds up and leads to the crash at time t_c, and finally a post crash period $[t_c, \infty]$. The separation of the periods $[t, t + h)$ and $[t + h, t_c)$ is conceptual. In reality, the mistake in $[t, t + h)$ may be made several times before the crash.

The first period $[0, t)$ is almost completely described above. Denote the information set during $[0, t)$ by $\mathcal{F}(\tau; \alpha, \beta)$, $\tau \in [0, t)$. The parameters α and β in $\mathcal{F}(\tau; \alpha, \beta)$ indicate that groups α and β exist in the market, are aware of each other, and form mean reversion expectations independently of each other, resulting in the distributions $p_\alpha(\lambda_i)$ and $p_\beta(\lambda_i)$. For simplicity, assume that during $[0, t)$, the means of the distributions are the same:

$$\lambda_a([0, t)) = \lambda_0 := \mathbb{E}_\alpha \lambda = \mathbb{E}_\beta \lambda.$$

This assumption can be relaxed without altering the argument. The market clearing condition is (7) and the equilibrium excess return is given by (2) as

$$\mathbb{E}(d \log S_t - \tilde{\mu} dt | \mathcal{F}(\tau; \alpha, \beta)) = \lambda_0 (\log \tilde{\vartheta}_t - \log S_t) dt. \tag{8}$$

FIGURE 1 ABOUT HERE.

3.2 Mean Reversion Illusion

For the second period $[t, t + h)$, consider the situation in Figure 1, where at time t a positive change in returns is observed. A-priori, investors in groups α and β expect reversion speed λ_0 in the mean ($\mathbb{E}_\alpha \lambda = \mathbb{E}_\beta \lambda = \lambda_0$).

Different reversion speeds λ_i correspond to different price levels $S(\tau, \lambda_i)$ after t. For example, consider time t^* in Figure 1 as the investment horizon. At this point, we have $S(t^*, \lambda_0) < S(t^*, \lambda_1) < S(t^*, \lambda_2)$ since $\lambda_0 > \lambda_1 > \lambda_2$.

In this period $[t, t + h)$, the difference in risk aversion between the groups becomes relevant. Group α hedges against a price level corresponding to faster reversion than λ_0 by replicating a put with strike at $S(t^*, \lambda_0)$ and maturity t^*. Then, group α manages its positions as if expecting a slow mean reversion of $\lambda < \lambda_2$. They take on higher risk (for example by holding or increasing long positions) and increase their chance to gain from unexpected slow reversion while at the same time being hedged against faster reversion than their a-priori expectation. If an investor buys an asset and simultaneously replicates a put on it, the net position of the purchase and the short sale from the replicating portfolio is positive (Lemma 3 in the Appendix). Therefore, the strategies of group α will exert less selling pressure after the positive change in returns than implied by the a-priori reversion expectations λ_0. Let λ_2 denote the expected mean reversion implied by group α's consolidated purchases and short sales. That is, λ_2 is the mean implied by the distribution of mean reversion expectations of group α post hedging:

$$\lambda_2 = \sum_{i \in \alpha} \lambda_i p_{\alpha_0 + \alpha_1}(\lambda_i),$$

where $\alpha =: \alpha_0 + \alpha_1$ denotes that group α now engages in dynamic hedging transactions (α_1) and outright spot market transactions (α_0). Denote the corresponding strategies by $\pi_{\alpha_0}(\lambda_i)$ and $\pi_{\alpha_1}(\lambda_i)$, respectively.

After the upward change in returns at time t, group β waits until $t + h$ to observe the market and does not engage in any transactions. Investors in group β know that the actual mean reversion will be determined by the expected mean reversion averaged over all market participants. Since these expectations cannot be observed directly, scrutinizing the market after a change in returns is a natural way to infer the expectations of the other participants. High selling pressure after a positive jump or high buying pressure after a negative jump indicates fast expected reversion. The investors in group β update their a-priori expectations with what they infer from observing the market, similar to a Bayesian update.

Let the information set in $[t, t+h)$ be denoted by $\mathcal{F}(\tau; \alpha_0)$, $\tau \in [t, t+h)$, where the parameter α_0 indicates that only group α is active on the market and that it cannot be observed that a fraction α_1 of their transactions is part of a dynamic hedge. The market clearing condition is

$$\sum_{i \in \alpha} (\pi_{\alpha_0}(\lambda_i) + \pi_{\alpha_1}(\lambda_i)) = 0.$$

The actual mean reversion is determined by group α alone:

$$\lambda_a([t, t+h)) = \lambda_2.$$

The equilibrium excess return is thus given by

$$\mathbb{E}(d \log S_t - \tilde{\mu} dt | \mathcal{F}(\tau; \alpha_0)) = \lambda_2 (\log \tilde{\vartheta}_t - \log S_t) dt.$$

The problem with the updating of group β's expectations of mean reversion is that only group α is active on the market. Group β mistakes the actual mean reversion λ_2 in $[t, t+h)$, which is the result of group α's strategies, to be representative of the mean reversion expectations of all market participants. Also, group β cannot tell from market transactions that a part of them are designed to mirror a put. If group α were to buy outright puts, or if they gave a public record of their dynamic hedging transactions, group β could calculate a put/call ratio to put the actual mean reversion λ_2 into perspective. Group β would recognize that the mean reversion expectations of the investors in the market are equal (or similar) to their own a-priori expectations and that they are hedged when taking riskier positions.

Without any information about the dynamic hedging, group β adjust their expectations to a slower mean reversion λ_2. They update the distribution of their mean reversion expectations to

$$\begin{aligned} p_{\beta_{ill}}(\lambda_i) &:= p_\beta(\lambda_i|\mathcal{F}(\tau;\alpha_0))) \\ &= w p_{\alpha_0+\alpha_1}(\lambda_i) + (1-w)p_\beta(\lambda_i), \end{aligned}$$

where $w \in [0,1]$ is a weight that describes how the agents in group β adapt their a-priori expectations to the ones they infer from the market, and $p_\beta(\lambda_i)$ is the a-priori distribution of group β. The probability distribution $p_{\beta_{ill}}(\lambda_i)$ has both a frequentist and a Bayesian interpretation: It describes the frequency with which the mean reversion expectation λ_i occurs in group β post adaptation. But because this frequency is the result of the adaptation process, the distribution also describes the beliefs of group β. The mean implied by $p_{\beta_{ill}}(\lambda_i)$ is

$$\lambda_1 := w\lambda_2 + (1-w)\lambda_0 < \lambda_0.$$

The result is a distribution of mean reversion expectations in the market that does not properly reflect the a-priori expectations of either group. This situation is called *mean reversion illusion* in this paper.

During the period $[t+h, t_c)$ group β becomes active in the market and actual mean reversion is determined by the average expectations across groups

$$\begin{aligned} \lambda_a([t+h, t_c)) &= \alpha\mathbb{E}_{\alpha_0+\alpha_1}\lambda + \beta\mathbb{E}_{\beta_{ill}}\lambda \\ &= \alpha\lambda_2 + \beta\lambda_1 < \lambda_0. \end{aligned}$$

Market participants in both groups implement strategies following mean reversion expectations that do not reflect their a-priori expectations. Group β is not aware of the fact that it adjusted its expectations to those of a risk tolerant minority. The market clearing condition is

$$\sum_{i\in\alpha}(\pi_{\alpha_0}(\lambda_i) + \pi_{\alpha_1}(\lambda_i)) + \sum_{i\in\beta}\pi_{\beta_{ill}}(\lambda_i) = 0,$$

and the excess return is given by

$$\mathbb{E}(d\log S_t - \tilde{\mu}dt|\mathcal{F}(\tau;\alpha_0,\beta_{ill}) = (\alpha\lambda_2 + \beta\lambda_1)(\log\tilde{\vartheta}_t - \log S_t)dt,$$

where $\mathcal{F}(\tau;\alpha_0,\beta_{ill})$ denotes the information set in $[t+h, t_c)$.

After a series of positive moves, the trajectory of the asset is in a position as shown in Figure 2. Investors have let the price move far above the level that represents the a-priori mean reversion expectations of the market.

FIGURE 2 ABOUT HERE.

If group β learned about their misconception in forming mean reversion expectations, they would immediately change their strategies to better reflect their a-priori expectations. This creates the crash potential, shaded black in Figure 2.

3.3 The Crash and its Aftermath

The mean reversion illusion is exposed when group β understands that it adapted its a-priori expectations of fast mean reversion to those of a more risk tolerant minority and that the a-priori expectations of that minority were even equal or similar to group β's own. If the hedge position of group α becomes public, the misunderstanding of group β is revealed.

Denote the information set that contains the disclosure of the hedge position and the size of groups α and β by $\mathcal{F}(\tau;\alpha_0,\alpha_1,\beta))$, $\tau\in[t_c,\infty]$. Upon learning this information set, group β adapts their mean reversion expectations and their strategies to $p_\beta(\lambda_i|\mathcal{F}(\tau;\alpha_0,\alpha_1,\beta))) = p_\beta(\lambda_i)$, their a-priori distribution (or a distribution very close to it). The new actual mean reversion is given by the new expectations

$$\begin{aligned}\lambda_a([t_c,\infty]) &= \alpha\mathbb{E}_{\alpha_0+\alpha_1}\lambda + \beta\mathbb{E}_\beta\lambda\\ &= \alpha\lambda_2 + \beta\lambda_0\\ &> \lambda_a([t+h,t_c)) = \alpha\lambda_2 + \beta\lambda_1.\end{aligned}$$

The excess returns are given by

$$\mathbb{E}(d\log S_t - \tilde{\mu}dt|\mathcal{F}(\tau;\alpha_0,\alpha_1,\beta)) = (\alpha\lambda_2 + \beta\lambda_0)(\log\tilde{\vartheta}_t - \log S_t)dt;$$

the market clearing condition is

$$\sum_{i\in\alpha}(\pi_{\alpha_0}(\lambda_i)+\pi_{\alpha_1}(\lambda_i))+\sum_{i\in\beta}\pi_\beta(\lambda_i)=0.$$

Table 1 shows a synopsis of the four periods, its filtrations, mean reversion expectations, and market clearing conditions.

TABLE 1 ABOUT HERE.

If group α disengages the dynamic hedging after the crash, having experienced its limited functionality in the situation of illiquidity,

$$\begin{aligned}\lambda_a([t_c,\infty]) &= \alpha\mathbb{E}_\alpha\lambda+\beta\mathbb{E}_\beta\lambda\\ &= \alpha\lambda_0+\beta\lambda_0=\lambda_0.\end{aligned}$$

In this case the information set, clearing condition, and excess returns are the same as in the first period $[0,t)$.

The disillusion, however, does not only entail the adaptation of the mean reversion parameter. Group β realizes that they are in a situation like the one sketched in Figure 2. That is, the price process followed a trajectory inconsistent with average a-priori mean reversion expectations from time $t+h$ on. Therefore, only correcting the mean reversion parameter is not sufficient. Rather, a correction in the price is necessary that sets it back onto the trajectory it would have taken if the mean reversion illusion had not occurred. This is the crash.

3.4 Mean Reversion Disillusion and October 19, 1987

If errors in the perception of mean reversion expectations played a role in the stock market crash of 1987, this would imply that there was an illusion and later a disillusion about the market's average a-priori mean reversion expectation. In the notation of Figure 2, we are looking for the points t and t_c and the related events. We cannot expect any particular news event to cause the illusion at time t, so it will be difficult to identify t. The point t_c is the point immediately before the crash. The disillusion is caused by a piece of information that is relevant for mean reversion expectations and that surprises the public. A mean reversion illusion is particularly likely to occur if hedges can be implemented that cannot be observed by other market participants. Therefore, we are searching for the disclosure of large hedge positions.

The three days prior to 19-Oct-1987 are of prime interest in this respect. From Wednesday, October 14, to Friday, October 16, the U.S. stock market lost more than ten percent. The Dow Jones Industrial Average fell from 2,508 at closing on Tuesday to 2,246 at closing on Friday, the S&P500 from 314 to 282 over the same time. The loss on Wednesday was three percent, on Thursday two percent, and on Friday five percent.

These drops can be attributed to fundamental reasons, namely to announcements concerning the simultaneous budget and trade balance deficits and to the House Ways and Means Committee's plans to eliminate tax benefits for takeovers. On Wednesday, October 14, the U.S. government announced that the trade deficit was about ten percent higher than expected. The dollar fell sharply in reaction, and this led to an expected decrease in foreign investment. At the same time, it became known that the House Ways and Means Committee filed legislation concerning takeovers (Brady et al. 1988, p. III-2f). Mitchell and Netter (1989) observed that the losses on the stock markets in reaction to these news items were largely confined to the U.S. market. This indicates that the losses where the result of revisions in fundamentals.

Portfolio insurance companies reacted by increasing their cash positions through sales of index futures. They sold 530 million dollars on Wednesday, 965 million dollars on Thursday, and 2.1 billion dollars on Friday, the latter being eleven percent of the total daily sales on the futures market (Brady et al. 1988, p. III-16). By the end of the week it became apparent to market participants that these sales were by far not sufficient to adjust the portfolio insurance positions adequately. The Brady Report mentions another eight billion dollars that were expected to be sold on the futures market. The implied volume of equities under portfolio insurance, 60 to 90 billion dollars, seems to have surprised the market. This may have been the event that disclosed the dynamic hedge positions and therefore the true a-priori mean reversion expectations of the market participants (Brady et al. 1988, p. 29).

The Brady Report explained the crash in part by the mere existence of portfolio insurance and associated program trading that cascaded in the crash. The view proposed here is quite different; it follows Black (1988). In this view, the unexpectedly high portfolio insurance volumes were *fundamental* information, not just a technical issue. They revealed that during the boom of 1987 a mean reversion illusion had occurred.

We are now in a position to formulate the hypothesis to be tested. Prior to the crash, we expect to see actual mean reversion slowing down from λ_0 (period 1) to $\alpha\lambda_2 + \beta\lambda_1 < \lambda_0$ (period 3). We do not expect to be able to discern the short period 2 when group β is observing the market. As mentioned before, the separation of periods 2 and 3 is notional. After the crash, we expect actual mean reversion to be faster ($\alpha\lambda_2 + \beta\lambda_0$, period 4) than before the crash.

4 Empirical Evidence of Mean Reversion Changes in the Crash of 1987

The data are daily closing prices of the S&P500 index between 4-Jan-1982 and 30-Dec-1991, a total of 2563 observations. The series was obtained from Datastream. All holidays that repeat the price of the previous day are deleted.

TABLE 2 ABOUT HERE.

4.1 Identification of Mean Reversion Illusion and Disillusion

We will consider the disillusion and the induced crash first. The hypothesis is that after the crash, we should see a faster mean reversion $\alpha\lambda_2 + \beta\lambda_0$ than before the crash, where it is $\alpha\lambda_2 + \beta\lambda_1$.

The observations 16-Oct-1987 to 26-Oct-1987 are eliminated from the returns and the price series of the S&P500. This excludes the crash itself and the large changes in returns that followed for a few days. These changes would interfere with and dominate the estimation of mean reversion. Model (2) is estimated for the 100, 200, ..., 1000 observations before and after the crash. (More precisely, it is estimated before and after the gap 16-Oct-1987 to 26-Oct-1987.) Table 2 reports the estimations.

The findings clearly support the hypothesis derived in Section 3. Up to 700 days before and after the crash, there is an increase in the estimated mean reversion parameter. For the segments of 100, 200, 400, and 500 days before and after the crash, the increase is significant according to a likelihood ratio test at a confidence level of 0.95 or higher. According to the t-statistics, all estimations of the post crash mean reversion parameter λ in the periods up to 700 days are significant on the two-sided 0.95 confidence level, and four out of seven are significant on the two-sided 0.99 confidence level. In summary, mean reversion clearly increased after the crash.

The time interval $[t, t + h)$ where the illusion sets in is more difficult to identify, of course. We search for a change in the mean reversion parameter from relatively high (λ_0) to relatively low ($\alpha\lambda_2 + \beta\lambda_1$) some time prior to the crash. According to the hypothesis, the segment with slower mean reversion $\alpha\lambda_2 + \beta\lambda_1$ should lead up to the crash.

The Brady Report locates the beginning of the bull market, which led up to the crash, in 1982. The contributing factors are described as *"... continuing deregulation of the financial markets; tax incentives for equity investing; stock retirements arising from mergers, leveraged buyouts and share repurchase programs; and an increasing tendency to include 'takeover premiums' in the valuation of a large number of* *stocks"* (Brady et al. 1988, p. 9, I-2). The level of the U.S. stock market by the end of 1986 is described as high, but not unprecedented in terms of price earnings ratios. The appreciation from January 1987 through August 1987, however, *"... challenged historical precedent and fundamental justification"* (Brady et al. 1988, p. 9, I-2).

Using this segmentation as a guideline, model (2) is estimated on the segments 2-Jan-1982 to 30-Dec-1986 and 2-Jan-1987 to 15-Oct-1987. That is, we set $t =$ 2-Jan-1987. Model (2) is estimated on the 1987 segment with the mean return fixed at the estimate from the period 1982–1986. Figure 3 illustrates the estimations. The estimate of the mean reversion parameter on the 1982–1986 segment is significant at the one-sided 0.95 confidence level. On the 1987 segment, mean reversion is negligible. Thus, following the segmentation suggested by the Brady Report, we find a change in mean reversion between the segments. We seek support for the segmentation suggested by the Brady Report using a statistically rigorous change point detector.

FIGURE 3 ABOUT HERE.

In order to detect a change in the parameter vector of model (2), a Generalized Likelihood Ratio (GLR) scheme is applied as a change point detector (Lai 1995). Let $S = \{S_t\}_{t\in\{1,\ldots,T\}}$ be the time series of index prices. The GLR scheme sets a change point at

$$\inf_{t\in\{1,\ldots,T\}} \left\{ \max_{1\leq k\leq t} \sup_{\theta\in\Theta} \left[\sum_{i=k}^{t} \log \frac{f_\theta(S_i|S_1,\ldots,S_{i-1})}{f_{\theta_0}(S_i|S_1,\ldots,S_{i-1})} \right] > c \right\}, \tag{9}$$

where T is the number of observations, Θ is the open parameter set, f_θ is the probability density given the parameter vector θ, θ_0 is the parameter vector of the null hypothesis, and c is an a-priori constant. There is no analytical expression or distribution result for the critical value c, so that it must be found by simulation methods. The problem is substantially simplified by the fact that we search for a single change point.

Problem (9) is decomposed into the following steps. On a baseline segment of the first m points of the series, model (2) is estimated, providing the null hypothesis $\hat{\theta}_0 = (\hat{\mu}_0,\, \hat{\lambda}_0,\, \hat{\sigma}_0)'$. Then, (2) is estimated on every single subseries $\{S_1,\ldots,\, S_j\}$, $j = m+1,\ldots,\, T$, resulting in a series of $\hat{\theta}_j$ maximizing the likelihood functions (6) of the subseries. From this series of parameter estimates, the probability densities $f_{\hat{\theta}_j}(S_j|S_1,\ldots,S_{j-1})$ for every $j = m+1,\ldots,\, T$ are obtained and

$$Z_j := \log \frac{f_{\hat{\theta}_j}(S_j|S_1,\ldots,S_{j-1})}{f_{\hat{\theta}_0}(S_j|S_1,\ldots,S_{j-1})}$$

is stored. From the resulting series $\{Z_j\}_{j\in\{m+1,\ldots,T\}}$, the statistics series

$$\xi_t = \max_{m+1\leq k\leq t} \sum_{j=k}^{t} Z_j,\ t = m+1,\ldots,T \tag{10}$$

is calculated. Because we search for a single change point, we can simply plot the $\{\xi_t\}$ series. Figure 4 shows the series for the observations 2-Jan-1986 through 15-Oct-1987 when the baseline distribution is estimated on the observations 2-Jan-1982 through 30-Dec-1985. The estimated mean is allowed to change with every single observation that is added.

The estimated parameters move away from the estimated baseline parameters at two distinct speeds as the sample size increases. This is the interpretation of the two trends in the series that can be distinguished in Figure 4. The trend break is at the turn of the years 1986 to 1987. This supports the segmentation suggested by the Brady Report and legitimates the ad-hoc method in Figure 3.

FIGURE 4 ABOUT HERE.

Critical values c for ξ_t are obtained in a simulation: 1,000 time series are generated according to model (2) with the parameters obtained from the estimation of the baseline sample period 2-Jan-1982 to 30-Dec-1985 (1,012 observations). The sample period 2-Jan-1986 through 15-Oct-1987, for which the detector series ξ_t in Figure 4 is plotted, consists of 454 observations. Therefore, for each of the 1,000 simulated time series we generate $1,012 + 454 = 1,466$ observations. On the first 1,012 observations of each series, model (2) is estimated. For each series the detector statistic ξ_t is calculated for the remaining 454 observations, yielding 454,000 observations of the detector statistic. The significance levels reported in Figure 4 are the quantiles of these 454,000 observations. By July 1987, the detector statistic for the S&P500 exceeds all common significance levels.

4.2 Simulations of the Size of the Crash

Given these estimations of the mean reversion illusion and using only data available on 16-Oct-1987, what would have been the estimate of the magnitude of a possible crash? More precisely, given that the mean reversion illusion occurred at the beginning of the year 1987, roughly 200 days before October 16, and given that the mean reversion disillusion occurs on October 16, what is the distance in the paths of the price series that must be offset? With respect to Figure 2, we are interested in the distance in the trajectories that is shaded black, measured at the point immediately before the crash. Note that we do not estimate the time of the crash, the disillusion is assumed to happen on October 16 for whatever reason.

To answer this question, model (2) is simulated with the estimated parameters reported in Figure 3. To simulate the year 1987, ten thousand paths of length 200 of model (2) under the parameter regime obtained from the 1982–1986 segment are generated, using the value 246.45 of the S&P500 on 2-Jan-1987 as the starting point. If a mean reversion illusion occurred in January 1987, it lasted for about 200 days up to 16-Oct-1987. Without the illusion, the process would have continued for another 200 days under the old regime. The simulation thus gives an estimate of the distribution of the index value $S_{\text{no illusion}}(200)$ on 16-Oct-1987 without mean reversion illusion. The actual value of the S&P500 at closing on 15-Oct-1987 was 298.08. The sample distribution of the difference $\log(S_{\text{no illusion}}(200)) - \log(298.08)$ is an estimate of the distribution of the magnitude of the crash.

Table 3 shows the sample distribution of the difference $\log(S_{\text{no illusion}}(200)) - \log(298.08)$ in Panel A. As expected, the distribution is negatively skewed. There is still a substantial probability for an upward jump, however, because even under the regime with stronger mean reversion there are a number of paths that end up above 298.08 after 200 days. The probability of a crash of minus 20 percent or more is greater than seven percent. The probability of a correction of minus ten percent or more is greater than 40 percent.

It seems very strict to fix the endpoint at 298.08. Therefore, we simulate model (2) for 10,000 sample paths under both parameter regimes, the 1982–1986 period ($S_{\text{no illusion}}$) and the 1987 period (S_{illusion}). Table 3 shows the sample distribution of the difference $\log(S_{\text{no illusion}}(200)) - \log(S_{\text{illusion}}(200))$ in Panel B. Even after only 200 days the difference in the mean reversion parameter λ results in substantial distances in the trajectories and thus substantial probabilities for large jumps when a mean reversion disillusion occurs. The probability of a crash of more than 20 percent is higher than 10 percent under this distribution.

TABLE 3 ABOUT HERE.

These sample distributions are calculated under the assumption that if the mean reversion illusion had not occurred, the path of the Brownian motion driving the return process could have been different from the one that materialized between 2-Jan-1987 and 15-Oct-1987. One might argue that the arrival of fundamental information that makes up the noise part would have been the same in either case. Under this assumption, we can reconstruct the white noise process between 2-Jan-1987 and 15-Oct-1987 from model (2) by

$$\hat{\varepsilon}_t = \frac{1}{\hat{\sigma}} \left[\hat{\mu} + \hat{\lambda} \log \hat{\vartheta}_t + (1 - \hat{\lambda}) \log S_t - \log S_{t-1} \right]$$

using the parameter estimates from the 1987 segment.

Plugging $\hat{\varepsilon}_t$ back into the model with the parameters from the 1982–1986 segment generates a point estimate for $S_{\text{no illusion}}(200)$ and thus a point estimate for the magnitude of the crash. According to this method, we have $S_{\text{no illusion}}(200) = 273.78$ and thus,

$$\log(S_{\text{no illusion}}(200)) - \log(298.08) = -0.085,$$

a correction of minus 8.5 percent.

Thus, we have three different forecasts of the magnitude of the stock market crash using only data available on October 16, 1987. A point forecast using the estimated white noise path from model (2) predicts a -8.5 percent correction. Two density forecasts, one with the endpoint of the price process fixed at the actual value on October 16 and one where this endpoint is simulated result in a 7.5 and in a 10 percent probability of a crash of 20 percent or more, respectively.

4.3 A Note on the Stock Market Crash of 1929 and Other Declines

The stock market crash of 1929 can not be explained by a mean reversion illusion and disillusion. The knowledge about the hedge portfolio of the Black-Scholes analysis was not available and it was not possible to implement mean reversion expectations in the same way as 1987. There were a number of coarser dynamic hedging strategies, for example stop loss orders, but these were easy to observe. Estimations of model (2) in analogy to Table 2 on the daily closing prices of the Dow Jones Industrial Average before and after 26-Oct-1929 show that there was no significant change in the mean reversion parameter before and after the crash. Similarly, the decline in internet stocks around the year 2000 is not accompanied by significant changes in mean reversion. These estimations are not reported for brevity.

The explanation of the 1987 crash offered here is the incorrect evaluation of a fundamental factor, mean reversion expectations. This fundamental factor refers to the behavior of other market participants and their expectations, not to the stock issuing companies. Many other declines in the stock market, for example the devaluation of internet and information technology stocks in 2000, can be attributed to an overvaluation of fundamentals that concern the earnings prospects of the stock issuing companies. While the decline is triggered in both cases by new information about the fundamental factor, there is one important difference: In the case of disclosure of mean reversion illusion, market participants realize a collective mistake in stock pricing that has to be corrected irrespectively of the situation of the issuing companies. It seems plausible that corrections like these can occur faster than those that involve business news, since a single news item can suffice.

5 Conclusions

Mistakes in the formation of mean reversion expectations could have caused the stock market crash of 1987. This view was proposed by Black (1988). It is closely related to the models proposed by Grossman (1988), Gennotte and Leland (1990), and Jacklin, Kleidon, and Pfleiderer (1992). These studies rejected the widely held view that the mere existence of portfolio insurance and cascading program trading caused the crash. Instead, they proposed that the disclosure of large hedge positions came as a surprise and as a piece of fundamental information to the market.

This study proposes a framework of heterogeneous agents who adapt their mean reversion expectations. We derive the testable hypothesis that according to this explanation of the crash, actual mean after the crash should be higher than before the crash. Also, immediately before the crash actual mean reversion should be low relative to periods not immediately prior to the crash.

Estimating an Ornstein-Uhlenbeck model for stock returns with mean reversion on daily data of the S&P500 index, the stock market crash of 1987 is examined in detail. Using the periods of the "sound" bull market 1982–1986 and the exaggeration in 1987 identified in the report of the Brady Commission, it is shown that mean reversion in 1987 was slower than during the period 1982–1986. A generalized likelihood ratio test shows a significant change in the parameters of the model in early 1987. This supports the segmentation in the Brady Report and the hypothesis of a mean reversion illusion.

After the crash in 1987, mean reversion was significantly stronger than before. This supports the hypothesis that a mean reversion disillusion had occurred. The cause of the disillusion can be identified as the disclosure of the surprisingly large volumes of equities under dynamic hedging during the week prior to the crash.

Simulations of the model with the estimated parameters of the two segments 1982–1986 and 1987 show that a crash of 20 percent or more had a probability of more than seven percent. A correction of minus 10 percent or more had a probability of more than 40 percent.

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Appendix

Lemma 1. *The expected value of the price process solving model (1) is given by* $\vartheta_t = S_0 e^{\mu t}$.

Proof. Rewrite (1) to

$$dS_t = (\mu - \lambda) S_t dt + \lambda \vartheta_t dt + \sigma S_t dW_t,$$

and solve the associated homogeneous equation

$$dX_t = (\mu - \lambda) X_t dt + \sigma X_t dW_t,$$

to obtain $X_t = \exp[(\mu - \lambda - \sigma^2/2)t + \sigma W_t]$. Then the solution to (1) is given by

$$\begin{aligned} S_t &= X_t \left(S_0 + \int_0^t (X_u)^{-1} \lambda \vartheta_u du \right) \\ &= S_0 \exp\left[\left(\mu - \lambda - \frac{\sigma^2}{2} \right) t + \sigma W_t \right] \left(1 + \lambda \int_0^t \exp\left[\left(\lambda + \frac{\sigma^2}{2} \right) u - \sigma W_u \right] du \right) \end{aligned}$$

Taking expectations, we obtain

$$\begin{aligned} \mathbb{E} S_t &= S_0 e^{(\mu - \lambda - \frac{\sigma^2}{2})\, t} \left(\mathbb{E} e^{\sigma W_t} + \lambda \int_0^t e^{(\lambda + \frac{\sigma^2}{2})\, u} \mathbb{E} e^{\sigma (W_t - W_u)} du. \right) \\ &= S_0 e^{\mu t - \lambda t} + S_0 e^{\mu t - \lambda t} \lambda \int_0^t e^{\lambda u} du \\ &= S_0 e^{\mu\, t}. \end{aligned}$$

□

Lemma 2. *Model (2) is a first-order approximation to model (1).*

Proof. The mean reversion term in the model (1) can be rewritten as

$$\lambda \frac{\vartheta_t - S_t}{S_t} dt = \lambda \left(\frac{\vartheta_t}{S_t} - 1 \right) dt.$$

Denote $r := \vartheta_t / S_t - 1$, then

$$1 + r = \frac{\vartheta_t}{S_t}$$

and as $\log(1+r) \doteq r$ we have a first-order equivalent representation

$$\lambda \frac{\vartheta_t - S_t}{S_t} dt \doteq \lambda \log \frac{\vartheta_t}{S_t} dt = \lambda(\log \vartheta_t - \log S_t) dt.$$

From Ito's Lemma, we have

$$d \log S_t = \frac{dS_t}{S_t} - \frac{\sigma^2}{2} dt.$$

Define $\tilde{\mu} = \mu - \sigma^2/2$ and $\tilde{\vartheta}_t := S_0 \exp(\tilde{\mu}\, t)$. Then there is a first-order equivalent of the model (1) given by (2):

$$\log S_t = \log S_0 + \tilde{\mu}\, t + \lambda \int_0^t (\log \tilde{\vartheta}_u - \log S_u) du + \sigma W_t.$$

□

Lemma 3. *The result of the market transactions of investors buying a stock and simultaneously replicating a put option on it is positive. That is, the purchases are greater than the sales.*

Proof. This will be shown here for the case of a European put option. According to the Black-Scholes model, the replicating portfolio of a European put on one share of the underlying stock consists of a short position of $|\Delta(t)|$. Δ is the sensitivity of the option to changes in the price of the underlying given by

$$\Delta(t) = \Phi(d_1) - 1 < 0,$$

$$d_1 = \frac{\log \frac{S}{X} + \left(r + \frac{1}{2}\sigma^2\right)(T-t)}{\sigma\sqrt{T-t}}.$$

Φ is the cumulative distribution function of the standard normal distribution, S is the stock price, X is the exercise price of the put option, r is the risk-free interest rate, $T-t$ is the time to maturity and σ^2 is the variance of the stock price.

The proceeds from the short position are invested and gain the risk-free interest rate r. Assume that the investor hedges every single stock that she buys. Her position $P(t)$ is (in terms of inventories)

$$P(t) = (S - S\Delta(t)e^{rt}) \cdot n,$$

where n denotes the number of shares. The assertion made here is equivalent to

$$\frac{1}{n} P(t) > 0 \quad \Longleftrightarrow \quad S > S\Delta(t)e^{rt}.$$

Now, it is obvious that

$$1 + e^{-rt} > \Phi(d_1),$$

as the exponential function is strictly positive on $\mathbb{R}$ and $\Phi(d_1) \in [0, 1]$ as it is a probability. It follows that

$$1 > (\Phi(d_1) - 1)\, e^{rt} \quad \Longrightarrow \quad 1 > \Delta(t) e^{rt}.$$

Multiplying with $S > 0$ proves the assertion. $\square$

Table 1: Four periods of different market conditions

Market characteristics around the 1987 stock market crash according to the explanation in Section 3. The entries for the fourth segment assume that group α does not disengage dynamic hedging after the crash. If they do, the entries are identical to those of period 1.

Interval	Mean Reversion	Information	Clearing	Excess Return
1. $[0,t)$ before illusion	$\lambda_a([0,t)) = \lambda_0$	$\mathcal{F}(\tau;\alpha,\beta)$	$\sum_{i\in\alpha}\pi(\lambda_i) + \sum_{i\in\beta}\pi(\lambda_i) = 0$	$\mathbb{E}(d\log S_t - \tilde{\mu}dt \mid \mathcal{F}(\tau;\alpha,\beta))$ $= \lambda_0(\log\tilde{\vartheta}_t - \log S_t)dt$
2. $[t,t+h)$ illusion sets in	$\lambda_a([t,t+h)) = \lambda_2$	$\mathcal{F}(\tau;\alpha_0)$	$\sum_{i\in\alpha}(\pi_{\alpha_0}(\lambda_i) + \pi_{\alpha_1}(\lambda_i)) = 0$	$\mathbb{E}(d\log S_t - \tilde{\mu}dt \mid \mathcal{F}(\tau;\alpha_0))$ $= \lambda_2(\log\tilde{\vartheta}_t - \log S_t)dt$
3. $[t+h,t_c)$ pre-crash	$\lambda_a([t+h,t_c))$ $= \alpha\lambda_2 + \beta\lambda_1$	$\mathcal{F}(\tau;\alpha_0,\beta_{ill})$	$\sum_{i\in\alpha}(\pi_{\alpha_0}(\lambda_i) + \pi_{\alpha_1}(\lambda_i))$ $+ \sum_{i\in\beta}\pi_{\beta_{ill}}(\lambda_i) = 0$	$\mathbb{E}(d\log S_t - \tilde{\mu}dt \mid \mathcal{F}(\tau;\alpha_0,\beta_{ill}))$ $= (\alpha\lambda_2 + \beta\lambda_1)(\log\tilde{\vartheta}_t - \log S_t)dt$
4. $[t_c,\infty]$ (post-) crash	$\lambda_a([t_c,\infty])$ $= \alpha\lambda_2 + \beta\lambda_0$	$\mathcal{F}(\tau;\alpha_0,\alpha_1,\beta)$	$\sum_{i\in\alpha}(\pi_{\alpha_0}(\lambda_i) + \pi_{\alpha_1}(\lambda_i))$ $+ \sum_{i\in\beta}\pi_{\beta}(\lambda_i) = 0$	$\mathbb{E}(d\log S_t - \tilde{\mu}dt \mid \mathcal{F}(\tau;\alpha_0,\alpha_1,\beta))$ $= (\alpha\lambda_2 + \beta\lambda_0)(\log\tilde{\vartheta}_t - \log S_t)dt$

Table 2: ESTIMATION OF MODEL (2) ON S&P500 BEFORE AND AFTER THE 1987 STOCK MARKET CRASH

Estimation of model (2) on sample periods before and after the 1987 stock market crash. The observations from 16-Oct-1987 through 26-Oct-1987 were deleted from the series. The numbers in parentheses are quasi-maximum-likelihood standard errors according to White (1982). The estimations of the mean returns and standard deviations are significant according to all common confidence levels with the single exception of the mean return of 100 days before the crash. Significance according to the 0.95 confidence level is marked by a single asterisk; the 0.99 confidence level is marked by a double asterisk.

	n days before Oct. 16, 1987			n days after Oct. 26, 1987			LR Test
n	$\hat{\mu}$	$\hat{\lambda}$	$\hat{\sigma}$	$\hat{\mu}$	$\hat{\lambda}$	$\hat{\sigma}$	$H_0 : \lambda_{\text{after}} = \lambda_{\text{before}}$
100	0.000483	**0.010565**	0.009632	0.001401	**0.16636****	0.016239	7.65**
	(0.000613)	(0.011637)	(0.000827)	(0.000114)	(0.061999)	(0.001703)	
200	0.001172	**0.005814**	0.010373	0.000851	**0.077153***	0.013997	6.19*
	(0.000401)	(0.005729)	(0.000581)	(7.6e-5)	(0.03205)	(0.001075)	
300	0.001004	**0.024993**	0.010127	0.000683	**0.052885***	0.012292	2.24
	(0.000133)	(0.015156)	(0.000592)	(4.9e-5)	(0.021259)	(0.000859)	
400	0.000672	**0.016753**	0.009907	0.000713	**0.052938****	0.011204	4.92*
	(0.000110)	(0.008603)	(0.000496)	(3.1e-5)	(0.019714)	(0.000722)	
500	0.001062	**0.013118**	0.009516	0.000769	**0.044117****	0.010935	5.52*
	(0.000103)	(0.008098)	(0.000425)	(2.9e-5)	(0.014459)	(0.000699)	
600	0.000834	**0.015902***	0.009034	0.000715	**0.033246****	0.01056	2.58
	(6.4e-5)	(0.006800)	(0.000379)	(2.8e-5)	(0.011698)	(0.000614)	
700	0.000951	**0.017962***	0.008701	0.000663	**0.023374***	0.010297	0.36
	(4.7e-5)	(0.007320)	(0.000344)	(3.2e-5)	(0.009914)	(0.000554)	
800	0.000801	**0.015205***	0.008523	0.000541	**0.008240**	0.010536	1.92
	(4.3e-5)	(0.0059002)	(0.000314)	(8.1e-5)	(0.005264)	(0.000496)	
900	0.000728	**0.010165***	0.008546	0.000563	**0.009867***	0.010475	0.00
	(5.1e-5)	(0.004539)	(0.000289)	(5.5e-5)	(0.005016)	(0.000451)	
1000	0.000578	**0.003738**	0.008442	0.000547	**0.009902***	0.010221	1.88
	(9.6e-5)	(0.002642)	(0.000269)	(4.5e-5)	(0.004796)	(0.000420)	

Table 3: SAMPLE DISTRIBUTIONS OF THE MAGNITUDE OF THE CRASH

Panel A shows the sample distribution of the difference $\log(S_{\text{no illusion}}(200)) - \log(298.08)$. 298.08 is the value of the S&P500 at closing on 15-Oct-1987. Panel B shows the sample distribution of the difference $\log(S_{\text{no illusion}}(200)) - \log(S_{\text{illusion}}(200))$ when 10,000 Brownian sample paths of length 200 are evaluated under both regimes, that of the 1982–1986 period ($S_{\text{no illusion}}$) and that of the 1987 period (S_{illusion}).

Panel A		Panel B	
r_i	$\mathbb{P}(r_i - 0.10 \leq r < r_i)$	r_i	$\mathbb{P}(r_i - 0.10 \leq r < r_i)$
		-0.5	0.0009
		-0.4	0.0053
-0.3	0.0029	-0.3	0.0221
-0.2	0.0753	-0.2	0.0751
-0.1	0.3652	-0.1	0.1572
0	0.4332	0	0.2333
0.1	0.1160	0.1	0.2297
0.2	0.0072	0.2	0.1687
0.3	0.0001	0.3	0.0775
		0.4	0.0244
		0.5	0.0052
		0.6	0.0006

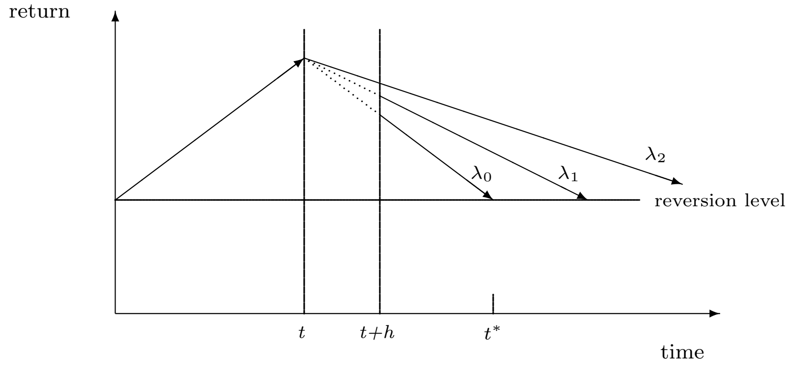

Figure 1: The development of mean reversion expectations.

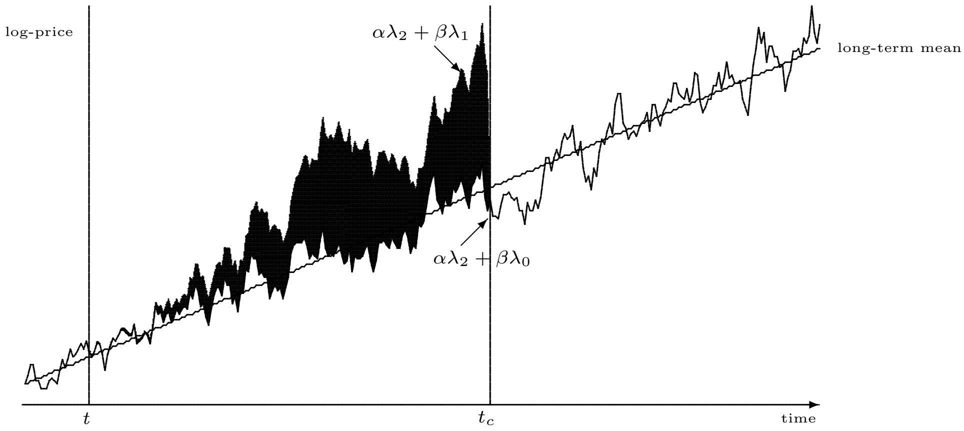

Figure 2: The mean reversion illusion between times t and t_c and the resulting difference in mean reversion speeds $\alpha\lambda_2 + \beta\lambda_1$ and $\alpha\lambda_2 + \beta\lambda_0$ drives the log price process above the $\alpha\lambda_2 + \beta\lambda_0$ level. The difference in the trajectories is shaded black and gives the potential crash at every point in time.

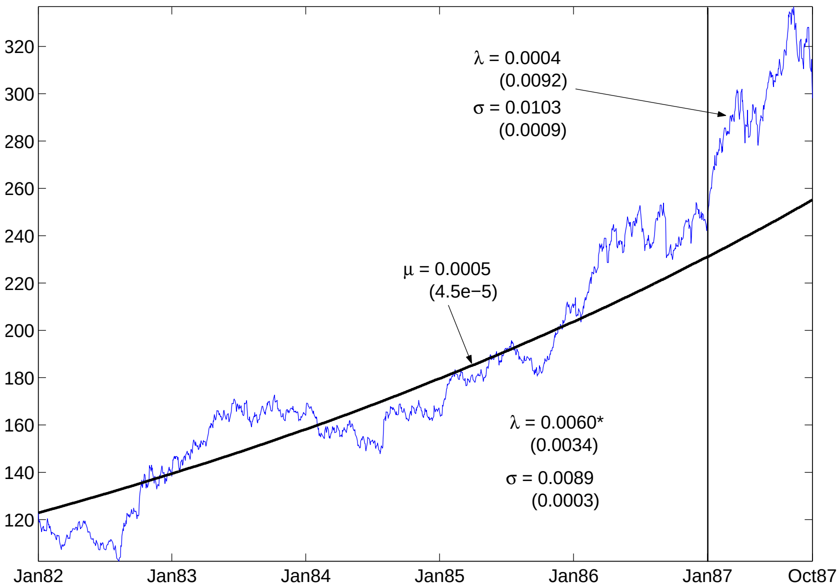

Figure 3: The bull market January 1982 to 15-Oct-1987 as seen in the S&P500. Using the segmentation of the Brady Report, we estimate model (2) on the period January 1982 to December 1986 and January 1987 to 15-Oct-1987. The numbers in parentheses are standard errors according to White (1982).

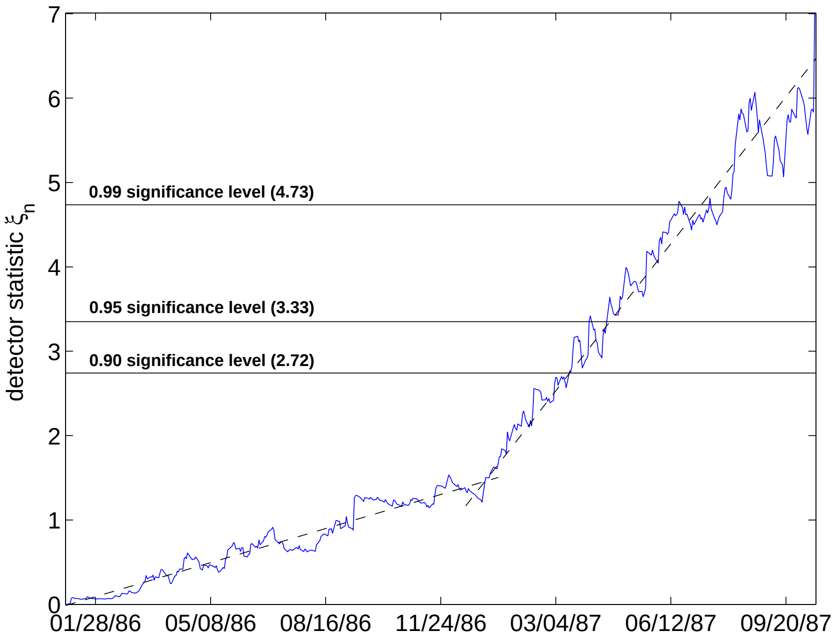

Figure 4: Change point detector statistic series $\{\xi_n\}$ given by Equation (10). The baseline parameter vector θ_0 was estimated on the segment 2-Jan-1982 through 30-Dec-1985. The detector statistics series is calculated and plotted for the observations 2-Jan-1986 through 15-Oct-1987. The significance levels are obtained by simulation of the statistic.