THE MODEL

$(\mathbb{R}, \mathcal{F}_t, \mathbb{P})$ probability space. $N(t)$ Gamma process with mean μ and variance ν. Increments

$$\Delta N(t,h) := N(t+h) - N(t).$$

Probability density

$$f_{\Delta N}(x) = \frac{x^{a-1}e^{-\frac{x}{b}}}{b^a\Gamma(a)}$$

where $a = \mu^2 h/\nu$ and $b = \nu/\mu$. Then

$$\mathbb{E}\Delta N(t,h) = \mu h, \qquad \mathbb{E}dN(t) = \mu dt,$$
$$\operatorname{Var}\Delta N(t,h) = \nu h, \qquad \operatorname{Var}(dN(t)) = \nu dt.$$

Setting $\mu = mf(t)$ and $\nu = \beta mf(t)$, we get

$$\mathbb{E}\Delta N(t,h) = m(F(t) - F(s))h, \qquad \mathbb{E}dN(t) = mf(t)dt,$$
$$\operatorname{Var}\Delta N(t,h) = \beta m(F(t) - F(s))h, \qquad \operatorname{Var}(dN(t)) = \beta mf(t)dt.$$

MAXIMUM LIKELIHOOD

Probability distribution of increments

$$N(t) - N(s) \sim \Gamma\left(\frac{m(F(t)-F(s))}{\beta}, \beta\right),$$

Parameter vector

$$\theta = (m,\, p,\, \beta) \qquad \Rightarrow \alpha(s,t) = \frac{m(F(t)-F(s))}{\beta}, \qquad \beta$$

Likelihood function

$$L(\theta|\{N(t)\}_{t\in\underline{T}}) = \prod_{t=1}^{T} \frac{1}{\beta^{\alpha(s,t)}\Gamma(\alpha(s,t))} x(t)^{\alpha(s,t)-1}\, e^{x(t)/\beta}.$$

$$x(t) = N(t) - N(t-1) \text{ (weeks)}$$

RELATED WORK

- De Vany and Walls 1996, De Vany: **Hollywood Economics** Routledge 2004, Elberse and Eliashberg 2003, Goetzmann et al. 2004, Neelamegham and Chintagunta 1999, Sawhney and Eliashberg 1996 **(Movie Revenues)**

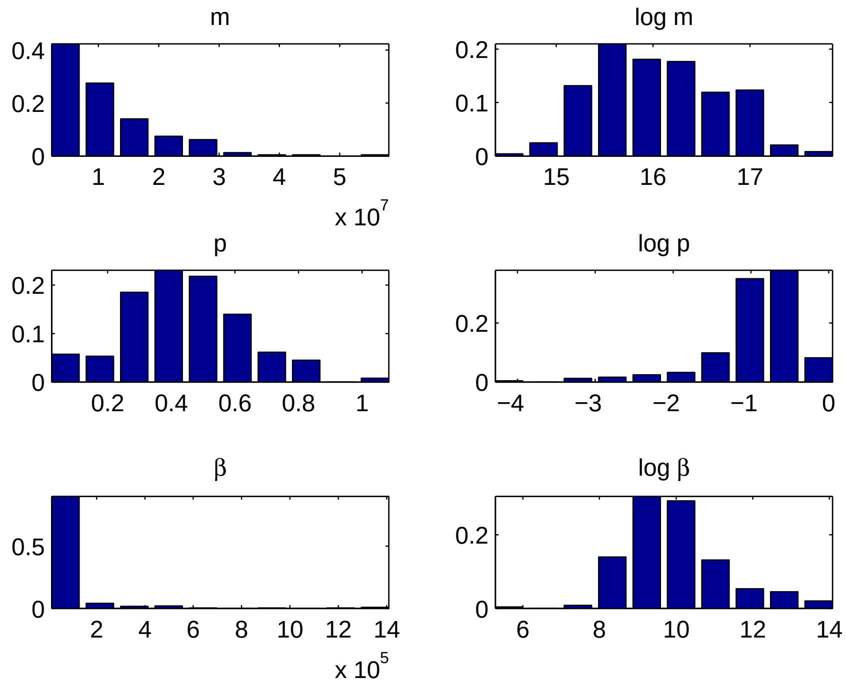
- Bass 1969, Boswijk and Franses 2004 **(Econometrics of Bass Model)**

- Todorov and Tauchen 2005, Tsilevich et al. 2001, Wenocur 1989 **(Gamma Process)**

FIRST STEP: MLE

For a database of earlier movies $i = 1, \ldots, N$, estimate $\widehat{\theta}_i$ by ML.

(100 most successful movies of 1998, 1999, and 2000)



SECOND STEP: REGRESSION

Log parameter estimates on movie characteristics

$$y = \begin{cases} \begin{bmatrix} \log \widehat{m}_1 \\ \vdots \\ \log \widehat{m}_N \end{bmatrix} \\ \\ \begin{bmatrix} \log \widehat{p}_1 \\ \vdots \\ \log \widehat{p}_N \end{bmatrix} \\ \\ \begin{bmatrix} \log \widehat{\beta}_1 \\ \vdots \\ \log \widehat{\beta}_N \end{bmatrix} \end{cases} \qquad X = \begin{bmatrix} \text{Genre} \\ \text{Initial Screens} \\ \text{Budget} \\ \text{Rating} \\ \text{Graphic} \end{bmatrix}$$

For movie under study: Plug in characteristics $\Rightarrow \widehat{\theta}_0$.

THIRD STEP: BAYESIAN UPDATE

Prior $f(\theta)$: from MLE step,

$$m, \beta \sim LN, \; p \sim N.$$

Likelihood of first observation

$$f(N(1) - N(0)|\theta) = \frac{1}{\beta^{\alpha(0,1)}\,\Gamma(\alpha(0,1))}\,\Delta N(1)^{\alpha(1)-1}\,e^{\frac{\Delta N(1)}{\beta}},$$

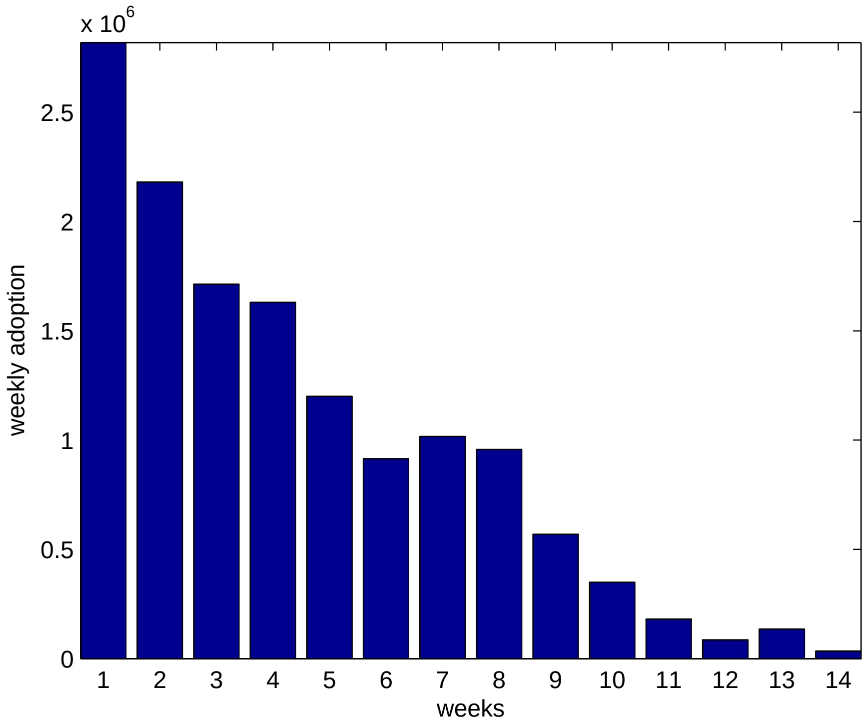
Update of $\widehat{\theta}_0$ as first observation arrives.

$$f(m|N(1) - N(0), \widehat{\theta}_0) = \int_p \int_\beta f(N(1) - N(0)|\theta) f(\theta) dp\, d\beta,$$

and so forth...

Issues: Non-linear, non-Gaussian $\Rightarrow$ MCMC (Metropolis-Hastings)

EXAMPLE: “THE OTHERS”, 2001



Initial Estimate: $\widehat{\theta}_0 = (6.4246e6, 0.2404, 41778)$.

EXAMPLE: “THE OTHERS”, 2001

Bayesian update via MCMC

	$\widehat{m}_0$	$\widehat{p}_0$	$\widehat{\beta}_0$
$\widehat{\theta}_0$	6.4249e6	0.2404	41778
update 1	1.1267e7	0.5113	191754
update 2	1.1934e7	0.4228	130523
update 3	1.2932e7	0.3335	81769
update 4	1.4161e7	0.2787	83688
update 5	1.5173e7	0.2398	48082
update 6	1.5500e7	0.2284	33965
update 7	1.6229e7	0.2076	38600
update 8	1.6973e7	0.1897	42804
update 9	1.7030e7	0.1883	35045
update 10	1.6647e7	0.1969	38163
update 11	1.6109e7	0.2108	51154
update 12	1.5615e7	0.2243	69292
update 13	1.5620e7	0.2259	64062
update 14	1.5408e7	0.2351	71788
MLE	1.6147e7	0.2367	51741

EXAMPLE: "THE OTHERS", 2001

Option valuation: Call, T2Mat = 12 weeks, $r = 3.41\%$, $a = \$5.65$.

Budget: \$ 27,000,000   U.S. gross revenue at $T = 14$: \$96,471,845

Expected revenue at $t = 0$ over life-time of option: Strike.

$$am(F(12) - F(0)) = \$5.65 \times 6,066,156 = 34,273,781$$

Option price at $t = 0$:

$$C(0,12) = 1,125,188.$$

At $T = 12$ gross revenue is \$95,471,845. Call expires \$61,198,000 in the money.

Miramax sells call on revenue: \$1,125,188 + (\$96,471,845 - \$61,198,060) $\approx$ \$8,300,000 net profit.

COMPARATIVE STATICS: THE "GREEKS"

- There is no Delta.
- $\partial C/\partial K$: negative, convex (same as standard)
- $\partial C/\partial \beta$ ("Vega"): positive, slightly concave (same as standard)
- $\partial C/\partial p$ positive, concave
- $\partial C/\partial m$ positive, near linear
- $\partial C/\partial r$ ("Rho"): negative, near linear (standard: positive)

CONCLUSION

1. Option pricing of non-decreasing process, here movie revenue. Uncertainty at release resolves quickly.

2. Specify gamma process as model and describe estimation:

   (a) Initial estimate with historical data.

   (b) Bayesian updating by MCMC.

3. Option price under physical measure.

4. Sensitivities to parameters.

5. Future research: Let revenue be traded...