Asymmetries, Breaks, and Long-Range Dependence in Realized Volatility

A Simultaneous Equations Approach

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Realized Volatility

- Leverage Effect (Black 1976). Regressions of the type

$$\frac{\hat{\sigma}_t - \hat{\sigma}_{t-1}}{\hat{\sigma}_{t-1}} = \beta r_{t-1} + \varepsilon_t$$

  yield highly significantly negative β's.
- Generalization of Leverage: Threshold Effects (Zakoian 1994, ...)
- Change-Points (Lamoureux and Lastrapes 1990,...)
- Volatility influences returns. (GARCH-in-mean, Engle, Lilien, and Robins 1984...)
- Long Memory (ABDL 2003, Martens, van Dijk, de Pooter 2004, Corsi 2004, ...)

Model

$$r_t = \beta' \mathbf{x}_t + \lambda v_t + \sigma_t e_t$$
$$v_t := (1-L)^d \log(\sigma_t) = g(\mathbf{z}_t;\xi) + \Theta_q(L) u_t,$$

- r_t returns, σ_t realized volatility, e_t, u_t white noise
- $\mathbf{x}_t = (1, \widetilde{\mathbf{x}}_t')' \in \mathbb{R}^{k_x+1}$ and $\widetilde{\mathbf{x}}_t$ is a vector of k_x explanatory variables,
- β is a $(k_x + 1)$-vector of parameters,
- $\lambda \in \mathbb{R}$ is the volatility-in-mean coefficient,
- d is fractional differencing parameter,
- $g(\mathbf{z}_t;\xi)$ is a general non-linear function of explanatory variables $\mathbf{z}_t$ and parameters ξ.
- $\Theta_q(L)$ MA lag polynomial.

Choices of g

$$r_t = \beta' \mathbf{x}_t + \lambda v_t + \sigma_t e_t$$
$$v_t = (1-L)^d \log(\sigma_t) = g(\mathbf{z}_t; \boldsymbol{\xi}) + \Theta_q(L) u_t$$

Example (Linear ARFIMA)

Set $\mathbf{z}_t = (v_{t-1}, \ldots, v_{t-p})'$.

$$g(\mathbf{z}_t; \boldsymbol{\xi}) := \phi_0 + \phi_1 v_{t-1} + \cdots + \phi_p v_{t-p}.$$

Volatility equation becomes:

$$\Phi_p(L)(1-L)^d \left[\log(\sigma_t) - \mu\right] = \Theta_q(L) u_t,$$

where $\Phi_p(L) = \left(1 - \phi_1 L - \phi_2 L^2 - \cdots - \phi_p L^p\right)$ and $\mu = \Phi_p^{-1}(1)\phi_0$. ARFIMA($p$,$d$,$q$) model (ABDL 2003).

Choices of g

$$r_t = \beta' \mathbf{x}_t + \lambda v_t + \sigma_t e_t$$
$$v_t = (1-L)^d \log(\sigma_t) = g(\mathbf{z}_t; \boldsymbol{\xi}) + \Theta_q(L) u_t$$

Example (STARFIMA, Lin and Teräsvirta 1994)

Set $\mathbf{z}_t = (\mathbf{w}_t', t)'$ where $\mathbf{w}_t = (v_{t-1}, \ldots, v_{t-p})'$.

$$g(\mathbf{z}_t; \boldsymbol{\xi}) := \phi_0 + \boldsymbol{\phi}' \mathbf{w}_t + \left(\widetilde{\phi}_0 + \widetilde{\boldsymbol{\phi}}' \mathbf{w}_t \right) f[\gamma(t-c)],$$

$f(y) = (1+e^{-y})^{-1}$ logistic function. Volatility equation becomes

$$(1-L)^d \log(\sigma_t) = \phi_0 + \sum_{i=1}^{p} \phi_i (1-L)^d \log(\sigma_{t-i}) + \left\{ \widetilde{\phi}_0 + \sum_{i=1}^{p} \widetilde{\phi}_i (1-L)^d \log(\sigma_{t-i}) \right\} f[\gamma(t-c)] + \Theta_q(L) u_t.$$

Choices of g

$$r_t = \beta' \mathbf{x}_t + \lambda v_t + \sigma_t e_t$$
$$v_t = (1-L)^d \log(\sigma_t) = g(\mathbf{z}_t; \boldsymbol{\xi}) + \Theta_q(L) u_t$$

Example (ARFIMA with leverage)

Let $\mathbf{z}_t = (\mathbf{w}_t', e_{t-1})'$. Accommodate leverage effects by

$$g(\mathbf{z}_t; \boldsymbol{\xi}) := \phi_0 + \boldsymbol{\phi}' \mathbf{w}_t + \left(\widetilde{\phi}_0 + \widetilde{\boldsymbol{\phi}}' \mathbf{w}_t \right) f(\gamma e_{t-1}).$$

(Glosten, Jagannathan, and Runkle 1993, van Dijk, Franses, and Paap 2002)

Choices of g

$r_t = \beta' \mathbf{x}_t + \lambda v_t + \sigma_t e_t$
$v_t = (1-L)^d \log(\sigma_t) = g(\mathbf{z}_t; \boldsymbol{\xi}) + \Theta_q(L) u_t$

Example (General Nonlinear ARFIMA)

Leave nonlinearity partially unspecified. Let $\mathbf{z}_t = (\mathbf{w}_t', \mathbf{s}_t')'$, $\mathbf{s}_t$ state variable (time, past returns, ...). MA order $q = 0$.

$$(1-L)^d \log(\sigma_t) = \nu_0 + \boldsymbol{\nu}' \mathbf{w}_t + \sum_{m=1}^{M} \nu_m f\left[\gamma_m \left(\boldsymbol{\omega}_m' \mathbf{z}_t - \eta_m\right)\right] + u_t.$$

$f(\cdot)$ logistic function, $\gamma_m > 0$ parameters, η_m location parameters, $\|\boldsymbol{\omega}_m\| = 1$ with

$$\omega_{m1} = \sqrt{1 - \sum_{j=2}^{q} \omega_{mj}^2}, \quad m = 1, \ldots, M.$$

(Teräsvirta, Lin, and Granger 1993, Medeiros, Teräsvirta, and Rech 2006)

Setup Linearity Test

$r_t = \beta' \mathbf{x}_t + \lambda v_t + \sigma_t e_t$
$v_t = (1-L)^d \log(\sigma_t) = g(\mathbf{z}_t; \boldsymbol{\xi}) + \Theta_q(L) u_t$

Volatility equation:

$$(1-L)^d \log(\sigma_t) = \nu_0 + \boldsymbol{\nu}' \mathbf{z}_t + \sum_{m=1}^{M} \nu_m f\left[\gamma_m \left(\boldsymbol{\omega}_m' \mathbf{z}_t - \eta_m\right)\right] + u_t.$$

Test $M = 0$ against $M > 0$. Null hypothesis:

$$\mathbb{H}_0 : \gamma_1 = \gamma_2 = \cdots = \gamma_M = 0.$$

Problem: Model not identified under null hypothesis. Standard inference not available.

Setup Linearity Test

$(1-L)^d \log(\sigma_t) = \nu_0 + \boldsymbol{\nu}'\mathbf{z}_t + \sum_{m=1}^{M} \nu_m f\left[\gamma_m\left(\boldsymbol{\omega}_m'\mathbf{z}_t - \eta_m\right)\right] + u_t$

Taylor-expand $f\left[\gamma_m\left(\boldsymbol{\omega}_m'\mathbf{z}_t - \eta_m\right)\right]$, $m = 1, \ldots, M$ around the null hypothesis $\mathbb{H}_0:\ \gamma_1 = \gamma_2 = \cdots = \gamma_M = 0$. $\mathbf{z}_t = (\mathbf{w}_t', \mathbf{s}_t')'$.

$$
\begin{aligned}
(1-L)^d \log(\sigma_t) = \pi_0 + \boldsymbol{\pi}'\mathbf{w}_t + \boldsymbol{\rho}'\mathbf{s}_t &+ \sum_{i=1}^{k_z}\sum_{j=i}^{k_z} \rho_{ij} z_{i,t} z_{j,t} \\
&+ \sum_{i=1}^{k_z}\sum_{j=i}^{k_z}\sum_{k=j}^{k_z} \rho_{ijk} z_{i,t} z_{j,t} z_{k,t} + u_t^*,
\end{aligned}
$$

$$
u_t^* = u_t + R_3(\mathbf{z}_t; \boldsymbol{\xi})
$$

New null hypothesis:

$$
\mathbb{H}_0:\ \rho = 0,\ \rho_{ij} = 0,\ \rho_{ijk} = 0.
$$

Test Statistic

$(1-L)^d \log(\sigma_t) = \nu_0 + \boldsymbol{\nu}'\mathbf{z}_t + \sum_{m=1}^{M} \nu_m f\left[\gamma_m\left(\boldsymbol{\omega}_m'\mathbf{z}_t - \eta_m\right)\right] + u_t$

Lagrange-Multiplier statistic

$$LM = \sum_{t=1}^{T} \widehat{\mathbf{q}}_t' \left\{ \sum_{t=1}^{T} \widehat{\mathbf{q}}_t \widehat{\mathbf{q}}_t' \right\}^{-1} \sum_{t=1}^{T} \widehat{\mathbf{q}}_t,$$

$\mathbf{q}_t$ gradient of the log-likelihood. Under the null hypothesis,

$$LM \xrightarrow{d} \chi^2_{k_z(k_z+1)/2 + k_z(k_z+1)(k_z+2)/6},$$

k_z dimension of $\mathbf{z}_t$.

Model Selection

$$r_t = \beta' \mathbf{x}_t + \lambda v_t + \sigma_t e_t$$
$$v_t = (1-L)^d \log(\sigma_t) = g(\mathbf{z}_t;\boldsymbol{\xi}) + \Theta_q(L)u_t$$

1. Set $g(\mathbf{z}_t;\boldsymbol{\xi}) = \xi_0 + \boldsymbol{\xi}'\mathbf{z}_t$.
2. Select elements of $\mathbf{x}_t$ and $\mathbf{z}_t$ by AIC or BIC.
3. Test null of linearity for Taylor-expansion of g.
4. Based on test results, estimate nonlinear specifications of g.

Estimation

$$r_t = \beta' \mathbf{x}_t + \lambda v_t + \sigma_t e_t$$
$$v_t = (1-L)^d \log(\sigma_t) = g(\mathbf{z}_t; \boldsymbol{\xi}) + \Theta_q(L) u_t$$

From distribution assumption,

$$\mathcal{L}_T(\psi) = \frac{1}{T} \sum_{t=1}^{T} \ell_t(\psi),$$

where

$$\ell_t(\psi) = -\frac{1}{2} \log(2\pi) - \frac{1}{2} \log(\sigma_u^2) - \frac{1}{2} \left(e_t^2 + u_t^2 \sigma_u^{-2} \right).$$

MLE of parameter vector:

$$\widehat{\psi} = \underset{\psi \in \boldsymbol{\Psi}}{\operatorname{argmax}} \mathcal{L}_T(\psi).$$

Under standard regularity conditions

$$\sqrt{T} \left(\widehat{\psi} - \psi^* \right) \xrightarrow{d} \mathrm{N} \left(\mathbf{0}, \mathbf{A}(\psi^*)^{-1} \mathbf{B}(\psi^*) \mathbf{A}(\psi^*)^{-1} \right).$$

Data

$$r_t = \beta' \mathbf{x}_t + \lambda v_t + \sigma_t e_t$$
$$v_t = (1 - L)^d \log(\sigma_t) = g(\mathbf{z}_t; \boldsymbol{\xi}) + \Theta_q(L) u_t$$

- Alcoa (AA), American International Group (AIG), Boeing (BA), Caterpillar (CAT), General Electric (GE), General Motors (GM), Hewlett Packard (HP), IBM, Intel (INTC), Johnson and Johnson (JNJ), Coca-Cola (KO), Merck (MRK), Microsoft (MSFT), Pfizer (PFE), Wal-Mart (WMT), Exxon (XON), from NYSE TAQ
- Sample period 3-Jan-1994 through 31-Dec-2003, 9.30am through 16.05pm. Estimation sample 3-Jan-1994 through 31-Dec-1999, forecast sample 2-Jan-2000 through 31-Dec-2003.
- 2-scale estimator of Aït-Sahalia, Mykland, and Zhang (2005).

Realized Volatility

$$r_t = \beta' \mathbf{x}_t + \lambda v_t + \sigma_t e_t$$
$$v_t = (1 - L)^d \log(\sigma_t) = g(\mathbf{z}_t; \boldsymbol{\xi}) + \Theta_q(L) u_t$$

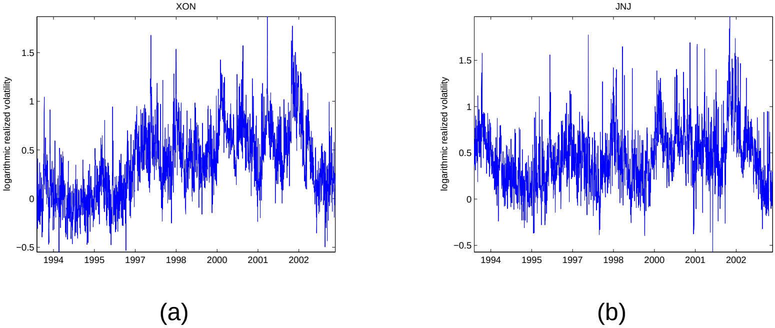

Figure: (a) Logarithmic Realized Volatility Exxon, (b) Logarithmic Realized Volatility Johnson and Johnson

Preliminary Analysis: Change Points

$$r_t = \beta' \mathbf{x}_t + \lambda v_t + \sigma_t e_t$$
$$v_t = (1-L)^d \log(\sigma_t) = g(\mathbf{z}_t; \boldsymbol{\xi}) + \Theta_q(L) u_t$$

Bai (1994, 1997) change point detector.

Series	Date	Stat	P-Value	Series	Date	Stat	P-Value
AA	n/a			INTC	4-Feb-1997	2.62	0.000
AIG	16-Jul-1998	2.47	0.000		20-Jul-1998	2.19	0.000
BA	8-Aug-1996	1.63	0.001	JNJ	n/a		
	15-Jul-1997	2.96	0.000	KO	22-Jul-1998	2.31	0.000
CAT	10-Jul-1997	2.96	0.000	MRK	21-Dec-1994	1.78	0.003
	3-Aug-1998	1.67	0.008		20-Jul-1998	2.05	0.000
GE	9-May-1997	1.98	0.001	MSFT	3-Dec-1996	2.78	0.000
GM	n/a				20-Jul-1998	1.73	0.005
HP	13-Apr-1995	2.08	0.000	PFE	3-Jul-1995	2.00	0.001
	1-Jul-1996	2.50	0.000	WMT	n/a		
IBM	20-Jul-1998	2.09	0.000	XON	1-Aug-1997	2.87	0.000

Preliminary Analysis: Leverage

$$r_t = \beta' \mathbf{x}_t + \lambda v_t + \sigma_t e_t$$
$$v_t = (1-L)^d \log(\sigma_t) = g(\mathbf{z}_t; \boldsymbol{\xi}) + \Theta_q(L) u_t$$

Black-(1976)-type regressions: $(\sigma_t - \sigma_{t-1})/\sigma_{t-1}$ on constant and returns r_{t-1}

Series	Constant	Return Coefficient
AA	0.0385(4e-5)	-0.0068(7e-6)
AIG	0.0403(4e-5)	-0.0099(1e-5)
BA	0.0567(1e-4)	-0.0093(3e-5)
CAT	0.0421(4e-5)	-0.0067(9e-6)
GE	0.0360(3e-5)	-0.0091(9e-6)
GM	0.0507(9e-5)	-0.0313(2e-5)
HP	0.0389(3e-5)	-0.0019(4e-6)
IBM	0.0381(3e-5)	-0.0061(7e-6)
INTC	0.0384(4e-5)	-0.0059(4e-6)
JNJ	0.0408(4e-5)	-0.0103(1e-5)
KO	0.0331(3e-5)	-0.0144(1e-5)
MRK	0.0401(4e-5)	-0.0050(1e-5)
MSFT	0.0445(4e-5)	-0.0073(7e-6)
PFE	0.0384(4e-5)	0.0017(9e-6)
WMT	0.0335(3e-5)	-0.0071(7e-6)
XON	0.0331(3e-5)	-0.0027(1e-5)
AVERAGE	0.0110(9e-6)	-0.0096(6e-6)

Preliminary Analysis: Long Memory

$$r_t = \beta' \mathbf{x}_t + \lambda v_t + \sigma_t e_t$$
$$v_t = (1-L)^d \log(\sigma_t) = g(\mathbf{z}_t; \boldsymbol{\xi}) + \Theta_q(L) u_t$$

GPH Estimates

Series	d_{GPH}
AA	0.74^{***}
AIG	0.75^{***}
BA	0.53^{***}
CAT	0.76^{***}
GE	0.71^{***}
GM	0.68^{***}
HP	0.64^{***}
IBM	0.58^{***}
INTC	0.55^{***}
JNJ	0.61^{***}
KO	0.72^{***}
MRK	0.60^{***}
MSFT	0.56^{***}
PFE	0.74^{***}
WMT	0.67^{***}
XON	0.61^{***}

Nonlinearities

$$r_t = \beta' \mathbf{x}_t + \lambda v_t + \sigma_t e_t$$
$$v_t = (1 - L)^d \log(\sigma_t) = g(\mathbf{z}_t; \boldsymbol{\xi}) + \Theta_q(L) u_t$$

Candidate variables for non-linear effects:

- Time (change-points, trends)
- Day-of-week
- Past returns (leverage)
- Macroeconomic announcements: FOMC meetings, Employment Situation Report (Bureau of Labor Statistics), CPI or PPI announcements

Model Selection

$r_t = \beta' \mathbf{x}_t + \lambda v_t + \sigma_t e_t$
$v_t = (1-L)^d \log(\sigma_t) = g(\mathbf{z}_t; \boldsymbol{\xi}) + \Theta_q(L) u_t$

Determine $\mathbf{x}_t$ and $\mathbf{z}_t$ by AIC; estimate model with $g(\mathbf{z}_t; \boldsymbol{\xi}) = \xi_0 + \boldsymbol{\xi}' \mathbf{z}_t$.

Series	$\widehat{\lambda}$	$\widehat{d}$	Weekdays (return)	Weekdays (volatility)	Announc. (return)	Announc. (volatility)	Leverage	Lags
AA	0.5381^{***}	0.0951	No	Yes	No	Yes	Yes	1
AIG	0.1482	0.2677^{**}	No	Yes	No	Yes	Yes	2
BA	0.1873^{***}	0.2118^{**}	No	Yes	No	Yes	Yes	2
CAT	0.2601	0.2851^{**}	No	Yes	No	Yes	Yes	2
GE	-0.6645^{***}	0.3928^{***}	No	Yes	No	Yes	Yes	3
GM	0.6743^{***}	0.2798^{***}	No	Yes	No	Yes	Yes	1
HP	-0.4455^{***}	0.2881^{***}	No	Yes	No	Yes	Yes	2
IBM	-0.2051	0.2363^{**}	No	Yes	No	Yes	Yes	2
INTC	-0.2667	0.4816^{***}	No	Yes	No	Yes	Yes	4
JNJ	-0.3733^{***}	0.2517^{**}	No	Yes	No	Yes	Yes	2
KO	-0.2217	0.3094^{***}	No	Yes	No	Yes	Yes	2
MRK	-0.2292	0.2398^{**}	No	Yes	No	Yes	Yes	2
MSFT	-0.0398	0.3675^{***}	No	Yes	No	Yes	Yes	3
PFE	-0.1752	0.2868^{**}	No	Yes	No	Yes	Yes	2
WMT	-1.0104^{***}	0.2699^{**}	No	Yes	No	Yes	Yes	2
XON	0.0821	0.3258^{***}	No	Yes	No	Yes	Yes	1

Linearity Test

$$r_t = \beta' \mathbf{x}_t + \lambda v_t + \sigma_t e_t$$
$$v_t = (1-L)^d \log(\sigma_t) = g(\mathbf{z}_t; \boldsymbol{\xi}) + \Theta_q(L) u_t$$

Pick candidates for non-linear variables by lowest p-value.

Series	Lags of v_t	Time	$\widehat{e}_{t-1}$	Lags of v_t and time	Lags of v_t and $\widehat{e}_{t-1}$	Time and $\widehat{e}_{t-1}$
AA	0.0178	0.6543	**0.0109**	0.0271	0.0212	0.1113
AIG	0.1754	0.1213	**0.0000**	0.0436	0.0067	0.0000
BA	0.1165	0.0456	**0.0000**	0.1265	0.0020	0.0000
CAT	0.0165	0.0564	**0.0000**	0.0465	0.0001	0.0000
GE	0.0345	0.4532	**0.0000**	0.0071	0.0002	0.0000
GM	0.0001	0.1134	**0.0000**	0.0000	0.0000	0.0001
HP	0.0217	0.1256	**0.0022**	0.0934	0.0178	0.0089
IBM	0.0131	0.0986	**0.0000**	0.0178	0.0001	0.0001
INTC	0.4563	0.0564	**0.0000**	0.0213	0.0001	0.0001
JNJ	0.0161	0.0089	0.0001	0.0005	0.0007	**0.0000**
KO	0.0041	0.0456	0.0019	0.0031	0.0003	**0.0001**
MRK	0.9167	0.0034	0.0003	0.0217	0.0323	**0.0000**
MSFT	0.1667	0.0674	**0.0000**	0.0548	0.0000	0.0000
PFE	0.1234	0.6753	0.0040	0.2763	**0.0021**	0.1081
WMT	0.5134	0.0134	0.0001	0.1247	0.0041	**0.0000**
XON	0.1234	0.0245	**0.0001**	0.1000	0.0145	0.0011

Estimation of Specific g

$r_t = \beta' \mathbf{x}_t + \lambda v_t + \sigma_t e_t$
$v_t = (1-L)^d \log(\sigma_t) = g(\mathbf{z}_t; \boldsymbol{\xi}) + \Theta_q(L) u_t$

We consider
1) Leverage and 2) Change points.
$\mathbf{z}_t = (\mathbf{w}_t', \mathbf{a}_t', s_t)'$ = (lags, announcements, state variable).
1) $s_t = e_{t-1}$, 2) $s_t = t$.

$$g(\mathbf{z}_t; \xi) = \phi_0 + \phi_1 v_{t-1} + \ldots \phi_p v_{t-p} + \nu' \mathbf{a}_t + (\tilde{\phi}_0 + \tilde{\phi}_1 v_{t-1} + \ldots \tilde{\phi}_p v_{t-p}) f(\gamma(s_t - c)).$$

Estimation of Specific g

$$r_t = \beta' \mathbf{x}_t + \lambda v_t + \sigma_t e_t$$
$$v_t = (1-L)^d \log(\sigma_t) = g(\mathbf{z}_t; \boldsymbol{\xi}) + \Theta_q(L) u_t$$

	Leverage Effect				Change-Points				
	d	c	γ	change	d	c	γ	date	change
AA	0.391	-0.699	$\gg 0$	0.073	0.396	1348	$\gg 0$	3-Mar-1999	-0.015
AIG	0.433	-2.70	1.33	0.325	0.474	936	$\gg 0$	2-Oct-1997	0.032
BA	0.351	-1.09	$\gg 0$	0.327	0.373	935	$\gg 0$	1-Oct-1997	0.023
CAT	0.389	-1.09	$\gg 0$	0.112	0.403	661	$\gg 0$	23-Aug-1996	0.012
GE	0.423	-4.98	0.770	0.215	0.456	1162	$\gg 0$	1-Sep-1998	0.078
GM	0.259	-1.03	$\gg 0$	0.088	0.258	1078	$\gg 0$	4-May-1998	-0.021
HP	0.438	-0.938	5.68	0.069	n/a	n/a	n/a	n/a	n/a
IBM	0.333	-0.969	$\gg 0$	0.157	0.363	811	$\gg 0$	4-Apr-1997	-0.013
INTC	0.363	-0.981	4.06	0.113	0.415	376	$\gg 0$	30-Jun-1995	0.062
JNJ	0.389	-2.45	1.45	0.161	n/a	n/a	n/a	n/a	n/a
KO	0.433	-1.708	2.55	0.393	n/a	n/a	n/a	n/a	n/a
MRK	0.382	-0.880	7.86	0.185	n/a	n/a	n/a	n/a	n/a
MSFT	0.359	-4.62	1.42	0.018	n/a	n/a	n/a	n/a	n/a
PFE	0.442	1.11	$\gg 0$	0.018	n/a	n/a	n/a	n/a	n/a
WMT	0.412	-2.57	1.03	0.172	n/a	n/a	n/a	n/a	n/a
XON	0.424	-0.903	$\gg 0$	0.092	n/a	n/a	n/a	n/a	n/a

Horse Race

$$r_t = \beta' \mathbf{x}_t + \lambda v_t + \sigma_t e_t$$
$$v_t = (1-L)^d \log(\sigma_t) = g(\mathbf{z}_t; \boldsymbol{\xi}) + \Theta_q(L) u_t$$

Mean Absolute Errors (MAE)

Series	ARFIMA	EWMA	HAR-RV	STARFIMA	FI-NN
AA	0.477	0.463	0.457	0.471	0.441
AIG	0.374	0.377	0.369	0.370	0.358
BA	0.406	0.418	0.397	0.416	0.379
CAT	0.416	0.417	0.406	0.419	0.393
GE	0.365	0.375	0.350	0.362	0.350
GM	0.394	0.396	0.384	0.393	0.365
HP	0.614	0.593	0.583	0.595	0.581
IBM	0.345	0.354	0.335	0.357	0.329
INTC	0.459	0.480	0.451	0.473	0.439
JNJ	0.383	0.390	0.374	0.378	0.357
KO	0.360	0.348	0.343	0.353	0.332
MRK	0.384	0.385	0.373	0.387	0.366
MSFT	0.367	0.384	0.358	0.372	0.355
PFE	0.442	0.433	0.426	0.434	0.424
WMT	0.417	0.414	0.407	0.416	0.400
XON	0.327	0.327	0.317	0.318	0.310

EWMA — exponentially weighted moving average

FI-NN — our model with Taylorized volatility equation

Conclusions

$$r_t = \beta' \mathbf{x}_t + \lambda v_t + \sigma_t e_t$$
$$v_t = (1-L)^d \log(\sigma_t) = g(\mathbf{z}_t; \boldsymbol{\xi}) + \Theta_q(L) u_t$$

1. Specify non-linear system of equations for returns and realized volatility.
2. In the presence of long memory, test the null of linearity of the volatility equation against non-linearity.
3. Application: Strong threshold effect, change-point evidence mixed.
4. Good forecasting performance.