Pricing an Option on Movie Revenue

Theory and Application

Don M. Chance[1] Eric Hillebrand[2] Jim E. Hilliard[1]

[1]Department of Finance
Louisiana State University

[2]Department of Economics
Louisiana State University

October 29, 2007 / UC Santa Barbara, CRFMS Seminar

Instruments on Entertainment Revenue

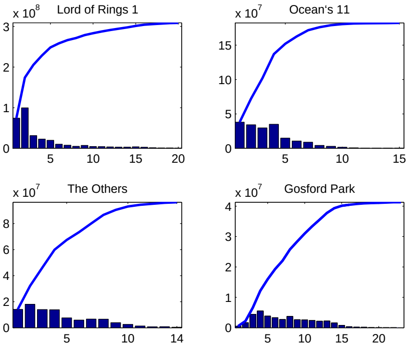
Contingent claims and related contracts based on entertainment revenues:

- 1992 Disney Eurobond paid interest based on a formula tied to revenues from a package of movies
- Ideas put forth for creation of an Entertainment Industry Options Exchange (Risk, 1997)
- Bowie Bonds
- Attempts to create an electronic market for derivatives on box office receipts (Risk, 2004)
- Hollywood Stock Exchange

Increasing interest in the statistical aspects of entertainment, particularly movies.

Movie Revenue

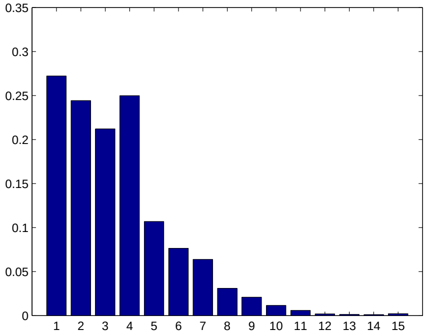
What are we trying to model?



Weekly and cumulative U.S. box office revenue for four example movies.

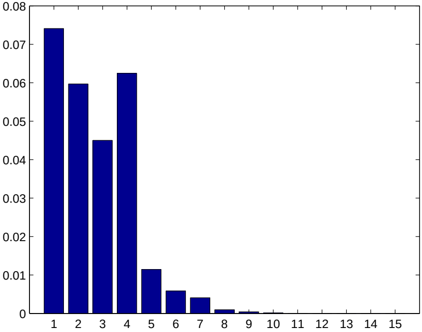
Movie Revenue

- Ocean's 11
- Returns as measured by reported budget of $140,000,000



Movie Revenue

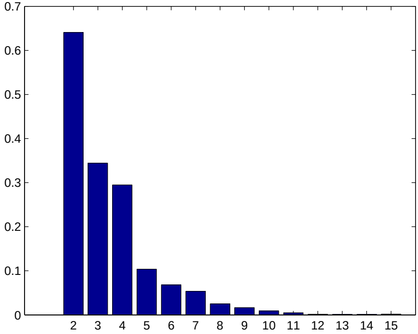
- Squared returns



Jump to Bass Model

Movie Revenue

- Log returns



Movie Revenue

- Characteristics:
  - Short life span
  - Non-negative increments, non-decreasing cumulative revenue
  - Risk is highly concentrated at the start and resolved quickly
- Objectives:
  - Specify revenue model
  - Estimate model parameters before and during release
  - Price European options on movie revenue

Bass Model

- We start with Bass' (1969) model for the adoption of an innovation.
- The hazard function $h(t)$ is

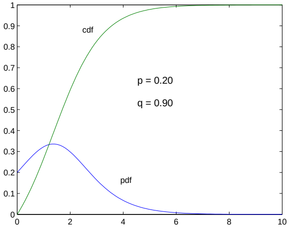
$$h(t) = \frac{f(t)}{1 - F(t)} = p + qF(t),$$

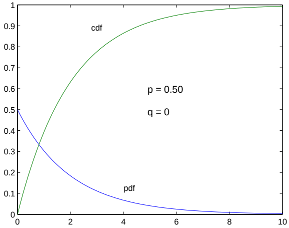
where

$$\begin{aligned} f(t) &= \text{cumulative adoptions with} F(0) = 0 \\ &= \frac{1-e^{-(p+q)t}}{1+e^{-(p+q)t} q/p} \\ p &= \text{coefficient of innovation} \\ q &= \text{coefficient of imitation} \end{aligned}$$

Bass Model

- Typical shapes of the Bass distribution





- Random variable is time to adoption for an individual

Bass Model

- Let m = maximum number of adopters (market potential).
- Under the Bass model, the adoption process is

$$n(t) = mF(t),$$

- such that the increment between times s and t is

$$n(t) - n(s) = m(F(t) - F(s)).$$

Bass Model Embedded in Gamma Process

- We model movie adoption as a Gamma process $N(t)$. The increments are Gamma-distributed such that

$$dN(t) \sim \text{Gamma}\left(\frac{m\,dF(t)}{\beta}, \beta\right).$$

- By the properties of the Gamma distribution, for arbitrary $s < t$,

$$\mathbb{E}(N(t) - N(s)) = m(F(t) - F(s)) = n(t) - n(s),$$

which calibrates the expected value to the Bass model.

Bass Model Embedded in Gamma Process

- Further,

$$\mathrm{Var}(N(t) - N(s)) = m\beta(F(t) - F(s)).$$

Jump to Return Slide

Related Work

- DeVany and Walls 1996, Elberse and Eliashberg 2003, Goetzmann et al. 2004, Neelamegham and Chintagunta 1999, Sawhney and Eliashberg 1996 (**Movie Revenue**)
- Bass 1969, Boswijk and Franses 2004 (**Bass Model and its Estimation**)
- Carr et al. 2006, Madan et al. 1990, 1991, 1998 (**Gamma Process**)

Setup

- Assumption: Movie revenue process uncorrelated with (stock) market risk.
- Start at time s, expiry at time T
- Cumulative revenue process $R(t) = aN(t)$, a average ticket price
- K strike price, $K_N := K/a$ strike price in adoption units.
- Call option payoff

$$\max\{0, R(T) - K\}$$

where $R(T)$ is revenue accumulated over the period $(0, T)$.

Pricing Formula

- $$C(s,T) = ae^{-r(T-s)}\mathbb{E}_s \max\{0, N(s) + U(s,T) - K_N\}$$

  where $U(s,T) = N(T) - N(s) \sim \text{Gamma}\left(\frac{m(F(T)-F(s))}{\beta}, \beta\right)$.

- $$C(s,T) = ae^{-r(T-s)} \int_d^\infty (N(s) + U(s,T) - K_N) g_U(u) du,$$

  where g gamma density and $d = \max\{0, K_N - N(s)\}$.

Pricing Formula

- $$C(s,T) = ae^{-r(T-s)}(m(F(T)-F(s))\Pi_1 + (N(s)-K_N)\Pi_2),$$

  where

- $$m(F(T)-F(s))\Pi_1 = \int_d^{\infty} u g_U(u) du$$
  $$= m(F(T)-F(s))\left(1 - \frac{\Gamma_{\frac{d}{\beta}}(\frac{m}{\beta}(F(T)-F(s))+1)}{\Gamma(\frac{m}{\beta}(F(T)-F(s)))+1)}\right),$$

  $\Gamma_d(\alpha) = \int_0^d t^{\alpha-1}e^{-t}dt$ incomplete Gamma function,

Pricing Formula

- and the probability to finish in the money

$$\Pi_2 = \int_d^\infty g_U(u)du = \mathbb{P}(U(s,T) > d)$$
$$= 1 - \frac{\Gamma_{\frac{d}{\beta}}(\frac{m}{\beta}(F(T) - F(s)))}{\Gamma(\frac{m}{\beta}(F(T) - F(s)))}.$$

- Similar to Bakshi and Madan (2002) catastrophe loss options.

Estimation

- Probability distribution of increments

$$N(t) - N(s) \sim \text{Gamma}\left(\frac{m(F(t) - F(s))}{\beta}, \beta\right),$$

- Parameter vector

$$\theta = (m, p, \beta),\ q \equiv 0.$$

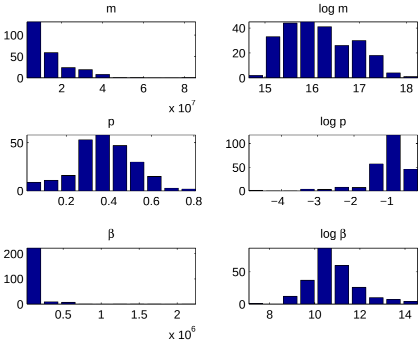
- Likelihood function

$$L(\theta | \{N(t)\}_{t \in \underline{T}}) = \prod_{t=1}^{T} \frac{1}{\beta^{\alpha}\Gamma(\alpha)} x(t)^{\alpha-1} e^{x(t)/\beta}.$$

$\alpha = m(F(t) - F(s))/\beta$, $x(t) = N(t) - N(t-1)$, t weeks

1st Step: MLE

- For a database of released movies $i = 1, \ldots, N$, estimate $\hat{\theta}_j$ by ML.
- We use 244 movies from 1998–2000.



2nd Step: Regression

- Log parameter estimates on movie characteristics.
- $$y = \left\{ \begin{bmatrix} \log \hat{m}_1 \\ \vdots \\ \log \hat{m}_N \end{bmatrix} \begin{bmatrix} \log \hat{p}_1 \\ \vdots \\ \log \hat{p}_N \end{bmatrix} \begin{bmatrix} \log \hat{\beta}_1 \\ \vdots \\ \log \hat{\beta}_N \end{bmatrix} \right. \qquad X = \begin{bmatrix} \text{Genre} \\ \text{Initial Screens} \\ \text{Budget} \\ \text{Rating} \\ \text{Graphic} \end{bmatrix}$$

2nd Step: Regression

- For movie under study: Plug in characteristics $\Longrightarrow \hat{\theta}_0$.

3rd Step: Bayesian Update

- Prior $f(\theta)$: From MLE step,

$$m, \beta \sim LN, \quad p \sim N.$$

- Likelihood of first observation [let $\alpha_1 = m(F(1) - F(0))/\beta$]

$$f(N(1) - N(0)|\theta) = \frac{1}{\beta^{\alpha_1}\Gamma(\alpha_1)} \Delta N(1)^{\alpha_1 - 1} e^{\frac{\Delta N(1)}{\beta}},$$

3rd Step: Bayesian Update

- Update of $\hat{\theta}_0$ as the first observation arrives: E.g.

$$f(m|N(1)-N(0),\hat{\theta}_0)=\int_p\int_\beta f(N(1)-N(0)|\theta)f(\theta)dp\,d\beta,$$

and so forth...

- Issues: Non-linear, non-Gaussian $\Longrightarrow$ MCMC (Metropolis-Hastings)

Example: “The Others,” 2001

- Initial estimate: $\hat{\theta}_0 = (1.0109e7, 0.2654, 92312)$.
- Bayesian update via MCMC

	$\hat{m}$	$\hat{p}$	$\hat{\beta}$
$\hat{\theta}_0$	1.0109e7	0.2654	92312
update 1	9.7506e6	0.5852	131455
update 2	1.1989e7	0.4947	349189
update 3	1.3707e7	0.3981	372646
...			
update 12	2.0891e7	0.2111	141012
update 13	2.0824e7	0.2141	134463
update 14	2.0624e7	0.2196	135265
MLE	2.0224e7	0.2200	108226

Example: "The Others," 2001

- European Call to price at $t = 0$
- Maturity at $T = 12$ weeks
- r=3.41%,
- a=$5.65
- Budget: $27,000,000
- U.S. gross revenue at end of lifetime ($t = 14$): $96,471,845

Example: “The Others,” 2001

- Set strike at expected revenue at $t = 0$ ($\hat{\theta}_0$):

$$am(F(12) - F(0)) = \$5.65 \times 6,066,156 = \$34,273,781$$

- Option price at $t = 0$:

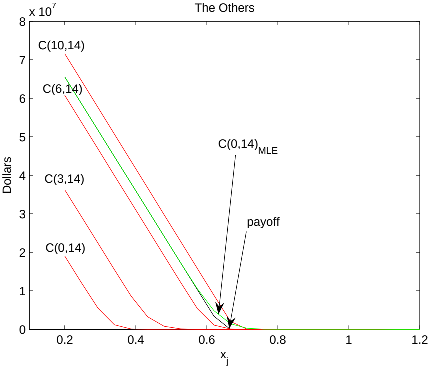
$$C(0,12) = \$1,125,188$$

- At $T = 12$, gross revenue is \$95,471,845.
- Call expires \$61,198,064 in-the-money.
- Miramax sold call on revenue at $t = 0$:

$$\$1,125,188 + (\$96,471,845 - \$61,198,060)$$
$$\approx \$8,300,000 \text{ net profit}$$

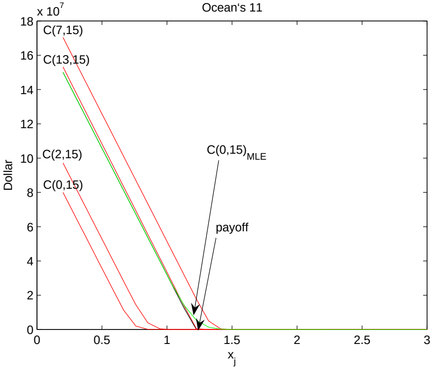
Example: “The Others,” 2001

- Option price profiles for updates



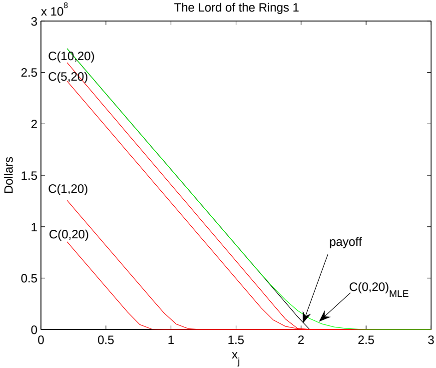
Example: "Ocean's 11," 2001

- Option price profiles for updates



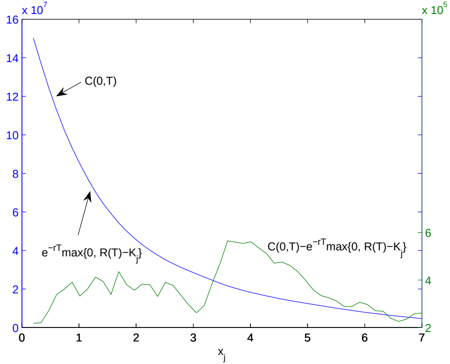
Example: “Fellowship of the Ring,” 2001

- Option price profiles for updates



Model Fit

- Option price profiles: model (MLE) and actual, averaged over 244 movies



Summary

- Movie revenue model: gamma process with Bass embedded
- Option price formula
- Estimation of parameters and Bayesian update

- Outlook
  - Platform releases: non-trivial q
  - Control problem, portfolio problem
  - Tradable revenue