

Neglecting Parameter Changes in Autoregressive Models

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INTRODUCTION

The estimation of autoregressive models is strongly biased if there are changes in the data generating parameters that shift the mean of the series. This phenomenon has been studied in the context of Dickey-Fuller tests [1,2,3]. We generalize these results to vector autoregressive data generating processes, shifts in the autoregressive parameters, and multiple shifts. We find that persistence is largely overestimated: the autoregressive coefficients grow close to one, autoregressive coefficient matrices display unit eigenvalues, and impulse response functions indicate spurious long-term influence. The theoretical results connect well to the change-point detection literature [4,5].

MODEL AND ASSUMPTIONS

The data generating process is

$$\mathbf{y}_t = \begin{cases} \mathbf{c}_1 + \Phi_1 \mathbf{y}_{t-1} + \varepsilon_t, & t \in \underline{T}_1 \\ \dots \\ \mathbf{c}_K + \Phi_K \mathbf{y}_{t-1} + \varepsilon_t, & t \in \underline{T}_K, \end{cases} \quad (1)$$

where $\mathbf{y}_t, \mathbf{y}_0 \in \mathbb{R}^N$, $\Phi_k \in \mathbb{R}^{N \times N}$, and $\underline{T}_k = \{T_{k-1} + 1, T_{k-1} + 2, \dots, T_k\}$.

Assumptions

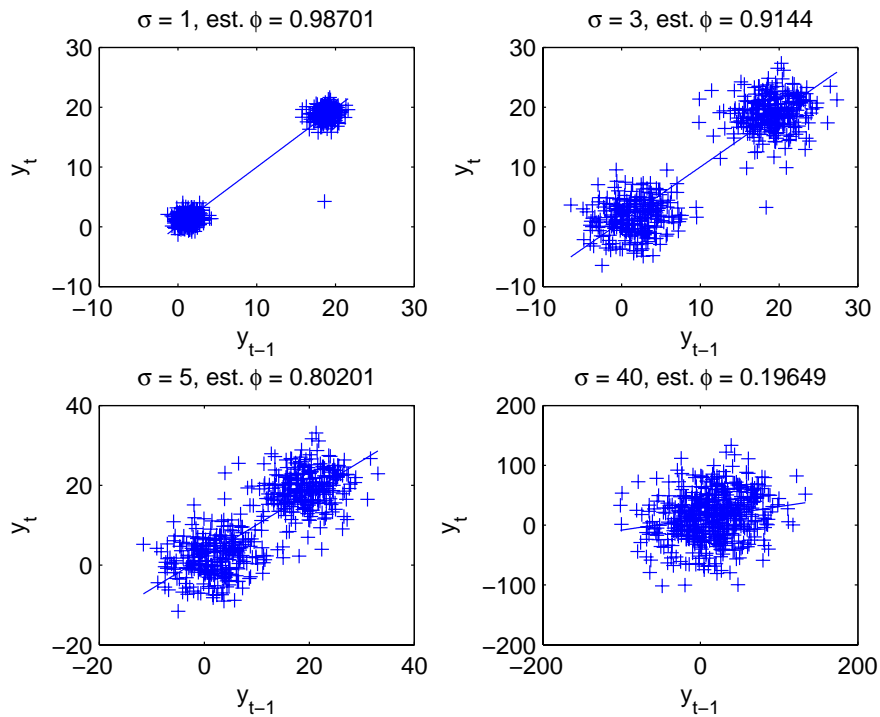
1. The stochastic driver $\{\varepsilon_t\}_{t \in \underline{T}}$ is i.i.d. vector white noise with mean zero and covariance matrix Ω .
2. The series is locally stationary, i.e. all eigenvalues of the Φ_k are inside the unit circle for all k . The local means are $\boldsymbol{\mu}_k = (\mathbf{I} - \Phi_k)^{-1} \mathbf{c}_k$.
3. The break fractions

$$\lambda_k := \frac{T_k - T_{k-1}}{T}$$

remain constant as the sample size T grows to infinity.

GEOMETRY OF THE PROBLEM

There is a trade-off between the size of the breaks and the variance of the stochastic driver of the process. (DGP AR(1) with $\phi = 0.20$, break occurs at $T/2 = 200$.)



THEOREM

VAR(1) is estimated globally on y_t and the parameter changes are ignored. Then, the least squares estimator satisfies in the probability limit

$$\begin{aligned}
 & \hat{\Phi} \left[\sum_{i,j \in \underline{K}, i \neq j} \lambda_i \lambda_j (\mu_i - \mu_j)(\mu_i - \mu_j)' \right. \\
 & \quad \left. + \lim_{T \rightarrow \infty} \frac{2}{T} \sum_{k=1}^K \sum_{t=T_{k-1}+1}^{T_k} \sum_{j=1}^{t-T_{k-1}-1} \Phi_k^{j-1} \Omega (\Phi_k^{j-1})' \right] \\
 & = \sum_{i,j \in \underline{K}, i \neq j} \lambda_i \lambda_j (\mu_i - \mu_j)(\mu_i - \mu_j)' \\
 & \quad + \lim_{T \rightarrow \infty} \frac{2}{T} \sum_{k=1}^K \Phi_k \sum_{t=T_{k-1}+1}^{T_k} \sum_{j=1}^{t-T_{k-1}-1} \Phi_k^{j-1} \Omega (\Phi_k^{j-1})'
 \end{aligned}$$

DISCUSSION

- In the scalar case, as $\sum_{i,j \in \underline{K}, i \neq j} \lambda_i \lambda_j (\mu_i - \mu_j)^2$ grows large relative to σ^2 , $\hat{\phi} \rightarrow 1$. As σ^2 grows large relative to $\sum_{i,j \in \underline{K}, i \neq j} \lambda_i \lambda_j (\mu_i - \mu_j)^2$,

$$\hat{\phi} \rightarrow \frac{\phi_1 \omega_1 + \phi_2 \omega_2 + \dots + \phi_K \omega_K}{\omega_1 + \omega_2 + \dots + \omega_K}, \quad \omega_k = \frac{\lambda_k}{1 - \phi_k^2}.$$

- AR(p) models can be written as VAR(1), so the Theorem applies. The result is that as $\sum_{i,j \in \underline{K}, i \neq j} \lambda_i \lambda_j (\mu_i - \mu_j)(\mu_i - \mu_j)'$ grows large,

$$\sum_{j=1}^p \hat{\phi} \rightarrow 1.$$

Similarly, for VAR(p), $\sum_{j=1}^p \hat{\Phi}_j$ displays a unit eigenvalue.

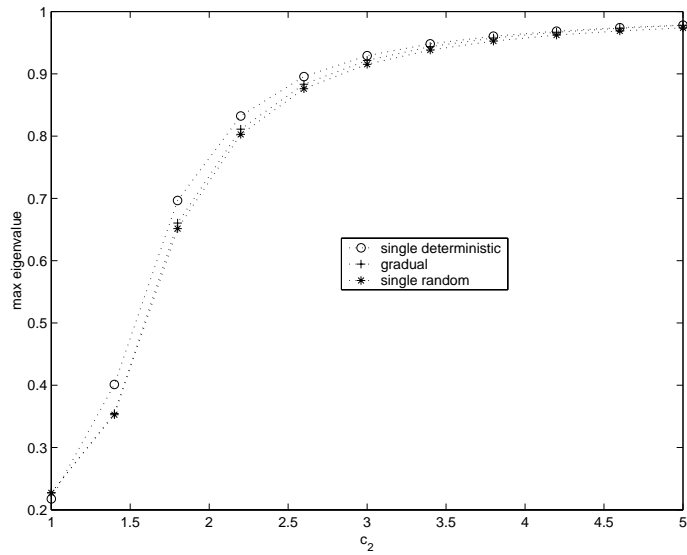
- In the case of a single break in a single component of the VAR(1), the eigenvector corresponding to eigenvalue 1 is the square of the change-point detector proposed in [4,5].

SIMULATIONS

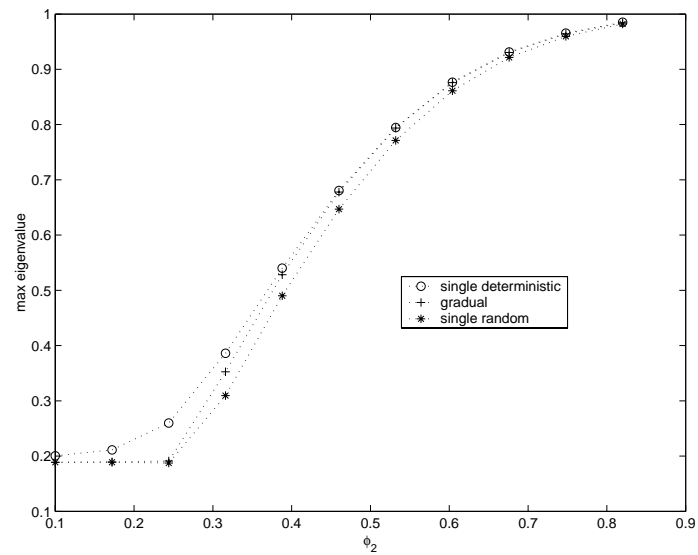
DGP: VAR(1), 11 different break sizes at $T/2$, single, gradual, and random breaks.

Largest eigenvalue in $\hat{\Phi}$ for

changes in c ,



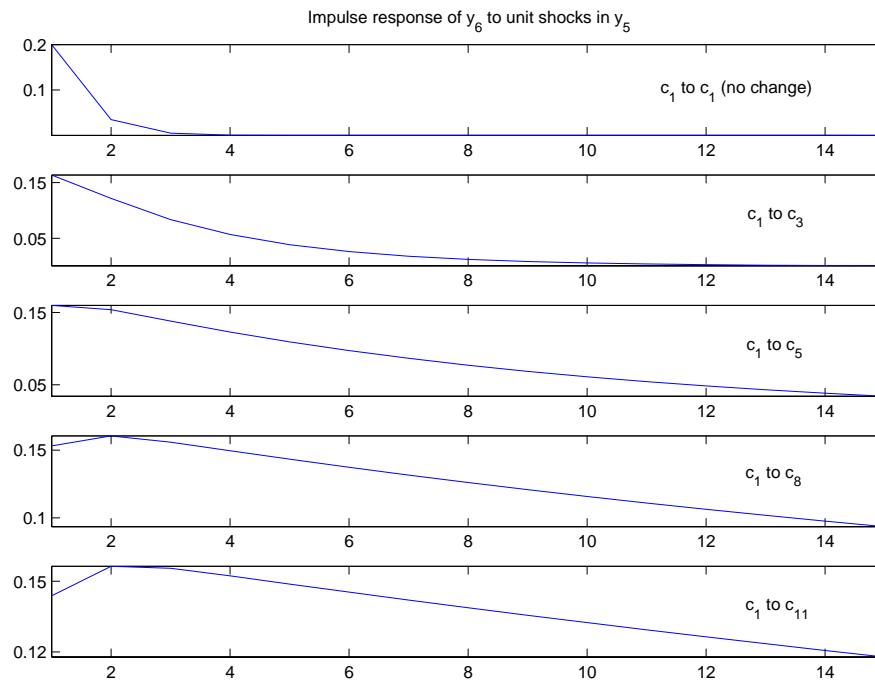
changes in Φ .



SIMULATIONS

DGP: VAR(1), 5 different single break sizes at $T/2$. The 6th entry in the VAR has lag 1 of the 5th entry in the DGP. Lag coefficient is 0.20.

Impulse response function of $y_{6,t}$ to unit shocks in $y_{5,t}$.



CONCLUSIONS

- Neglected parameter changes lead to overestimation of persistence in AR models.
- AR(1): $\hat{\phi} \rightarrow 1$; AR(p): $\sum \hat{\phi}_j \rightarrow 1$; VAR(1): $\hat{\Phi}$ has unit eigenvalue, VAR(p): $\sum \hat{\Phi}_j$ has unit eigenvalue. Impulse response functions are over-estimated.

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