This appendix contains a more detailed description of functional forms and the solution technique used in our simulations. We begin by choosing the following Cobb-Douglas form of the utility function:

\[ U(x, y) = \varepsilon \log x + (1 - \varepsilon) \log y, \quad 0 \leq \varepsilon \leq 1 \]

where \( x \) = domestic, \( y \) = foreign good. We can vary the weights \( \varepsilon, \left(1 - \varepsilon\right) \) to show how the economy’s response to a world interest rate shock is affected by how “open”, as measured by \( (1 - \varepsilon) \), it is. The partial derivatives of \( U(x, y) \) are given by:

\[
U_x = \varepsilon / x, \quad U_{xx} = -\varepsilon / x^2, \quad U_y = (1 - \varepsilon) / y, \quad U_{yy} = -(1 - \varepsilon) / y^2, \quad U_{xy} = 0
\]

Likewise the production function is specified as:

\[ F(k) = k^\gamma, \quad 0 < \gamma < 1, \quad F'(k) = \gamma k^{\gamma - 1} > 0, \quad F''(k) = -(1 - \gamma) k^{\gamma - 1} < 0 \]

where \( k \) = capital stock. While the simulations reported in the paper specify a value of \( \gamma \) equal to 0.3, we experimented with values of \( \gamma \) ranging from 0.2 to 0.5.

For the cost of adjustment investment function, we choose a standard quadratic specification:

\[ J(I) = I \left[ 1 + \frac{I}{2} \right], \quad J'(I) = 1 + I, \quad J''(I) = 1, \quad J(0) = 1, \quad J'(0) = 1 \]

where \( I \) = investment.
Similarly, we choose a quadratic version of the net export function:

\[ Z = Z(\sigma) = \frac{1}{2} \sigma^2, \quad Z' = \sigma \]  

(4)

where \( \sigma \) = relative price of the foreign in terms of the domestic good. Note that this specification of \( Z \) also pins-down the parameter \( \beta \), (which we use in the expressions for \( a_{41}, a_{42}, a_{43} \)). It is equal to:

\[ \beta = Z'(\sigma) - \frac{Z(\sigma)}{\sigma} = \frac{\sigma}{2} \]  

(5)

Finally, the debt function, (equation (1) of our paper), is also specified as quadratic:

\[ r = r(b) = r^* + \frac{b^2}{2}, \quad \alpha' = b, \quad \alpha'' = 1 \]  

(6)

6. Steady State

In calculating the impact of world interest rate shocks, we must solve for the steady state equilibrium, equation system (9) in the paper. Given the above specification, it is rewritten as:

\[ \frac{\varepsilon}{\tilde{x}} = \frac{\tilde{\lambda}}{\tilde{\sigma}} \]  

(9a)

\[ \frac{(1 - \varepsilon)}{\tilde{y}} = \tilde{\lambda} \]  

(9b)

\[ \tilde{q} = 1 \]  

(9c)
The steady state values of \( k \) and \( b \) are determined by the condition (9e) and are independent of the parameter \( \varepsilon \), (remember that \( \delta \) is the exogenous rate of time preference). The variables that are influenced by \( \varepsilon \) are the steady state values of \( x \), \( y \), \( \lambda \), and \( \sigma \). Given the steady state values of \( k \) and \( b \), the steady state values of \( x \), \( y \), \( \lambda \), and \( \sigma \) are found using equations (9a), (9b), (9d) and (9f). Of course, a value of \( \delta \) greater than \( r^* \) must specified to calculate the steady state equilibrium.

Substitution reduces (9a), (9b), (9d) and (9f) to two equations. We use then use a fortran program which calls the IMSL routine DNEQNF to solve the remaining system of nonlinear equations. Table 1 of our paper reports the steady state values that solve this system of equations for several values of \( \varepsilon \) and \( r^* \). By downloading the fortran routine “solve.f”, the reader can try alternative parameter values for the simulations.

We next state for your information some of the important mathematical expressions of the paper using the above parameterizations.

**The Partial Derivatives of Equation (7)**

Next, we state the partial derivatives of equation system (7) given our parameterization.
For $x$:

\[
\frac{dx}{d\lambda} = \frac{-\tilde{\sigma}\tilde{x}^2}{\varepsilon(\tilde{\sigma}^2 + \tilde{x})} < 0, \quad \frac{dx}{dq} = \frac{\tilde{x}}{\tilde{\sigma}^2 + \tilde{x}} > 0, \quad \frac{dx}{dk} = \frac{-\delta\tilde{x}}{\tilde{\sigma}^2 + \tilde{x}} < 0
\]

For $y$:

\[
\frac{dy}{d\lambda} = \frac{-\tilde{y}^2}{(1 - \varepsilon)} < 0, \quad \frac{dy}{dq} = \frac{dy}{dk} = 0
\]

For $\sigma$:

\[
\frac{d\sigma}{d\lambda} = \frac{\tilde{x}^2}{\varepsilon(\tilde{\sigma}^2 + \tilde{x})} > 0, \quad \frac{d\sigma}{dq} = \frac{-\tilde{\sigma}}{\tilde{\sigma}^2 + \tilde{x}} < 0, \quad \frac{d\sigma}{dk} = \frac{\delta\tilde{\sigma}}{\tilde{\sigma}^2 + \tilde{x}} > 0
\]

The Elements of the Coefficient Matrix of Equation (10)

We next state the expressions for the $a_{ij}$ elements of the linearized coefficient matrix (10), which we need to calculate the eigenvalues. We must calculate the eigenvalues, because they appear in the expressions for the dynamic responses of the economy to a permanent world interest shock and in expression for the effect on welfare.

\[
a_{11} = 0, \quad a_{12} = 0, \quad a_{13} = 0, \quad a_{14} = -\tilde{\lambda} \tilde{b} < 0
\]

\[
a_{21} = 0, \quad a_{22} = \delta, \quad a_{23} = \frac{(1 - \gamma)\gamma\tilde{k}\gamma^2(\tilde{\sigma}^2 + \tilde{x})}{\tilde{\sigma}^2 + \tilde{x} + 1} > 0, \quad a_{24} = \frac{\varepsilon(\tilde{\sigma}^2 + \tilde{x}) + \tilde{x}^2\tilde{\lambda}}{\varepsilon(\tilde{\sigma}^2 + \tilde{x} + 1)} > 0
\]
\[ a_{31} = 0, \quad a_{32} = 1, \quad a_{33} = 0, \quad a_{34} = 0 \]

\[
a_{41} = -\frac{2\varepsilon(\bar{\sigma}^2 + \bar{x})\bar{y}^2 + (1 - \varepsilon)\bar{x}^2}{2(1 - \varepsilon)(\bar{\sigma}^2 + \bar{x})} < 0, \quad a_{42} = \frac{\bar{\sigma}}{2(\bar{\sigma}^2 + \bar{x})} > 0
\]

\[
a_{43} = -\frac{-\delta\bar{\sigma}}{2(\bar{\sigma}^2 + \bar{x})} < 0, \quad a_{44} = \delta + \bar{b}^2 > 0
\]

The values of these expressions, and, thus, the eigenvalues, change with the parameter \(\varepsilon\).

**The Steady State Comparative Statics**

Here we state the steady state comparative statics, equations (A3.a-h) of the paper, employing our parameterization:

\[
\frac{d\tilde{b}}{dr^*} = -\frac{1}{\tilde{b}}, \quad \frac{d\tilde{\kappa}}{dr^*} = \frac{d\bar{\sigma}}{dr^*} = 0
\]

\[
\frac{d\tilde{\sigma}}{dr^*} = -\frac{2\delta(1 - \varepsilon)\bar{x}^2}{\tilde{b}\left[(1 - \varepsilon)\bar{x}^2 + 2\varepsilon(\bar{\sigma}^2 + \bar{x})\bar{y}^2\right]} < 0, \quad \frac{d\tilde{\lambda}}{dr^*} = -\frac{-2\delta(1 - \varepsilon)\varepsilon(\bar{\sigma}^2 + \bar{x})}{\tilde{b}\left[(1 - \varepsilon)\bar{x}^2 + 2\varepsilon(\bar{\sigma}^2 + \bar{x})\bar{y}^2\right]} < 0
\]

\[
\frac{d\bar{x}}{dr^*} = -\bar{\sigma} \frac{d\bar{\sigma}}{dr^*} > 0, \quad \frac{d\bar{y}}{dr^*} = \frac{2\delta\varepsilon(\bar{\sigma}^2 + \bar{x})\bar{y}^2}{\tilde{b}\left[(1 - \varepsilon)\bar{x}^2 + 2\varepsilon(\bar{\sigma}^2 + \bar{x})\bar{y}^2\right]} > 0
\]

\[
\frac{d\tilde{\tilde{b}}}{dr^*} = \delta \left[\frac{\tilde{b}}{dr^*} + \bar{\sigma} \frac{d\tilde{b}}{dr^*}\right] < 0, \quad \frac{d\tilde{\tilde{b}}^*}{dr^*} = -\frac{-\delta}{\tilde{b}} < 0
\]
where \( T_b \) = trade balance in terms of the domestic good and \( T_b^* \) is the trade balance in terms of the foreign good.

The Initial Dynamics

1. Debt

\[
\frac{d\dot{b}(0)}{dr^*} = \frac{1}{\bar{b}(\mu_1 + \mu_2 - \delta)} \left[ (\mu_1^2 + \mu_2 \mu_1 + \mu_2^2) - \delta (\mu_1 + \mu_2) - a_{23} \right]
\]

2. Capital

\[
\frac{d\dot{k}(0)}{dr^*} = \frac{a_{24}}{\bar{b}(\mu_1 + \mu_2 - \delta)}
\]

3. Shadow Value of Capital

a. Initial Jump

\[
\frac{dq(0)}{dr^*} = \frac{a_{24}}{\bar{b}(\mu_1 + \mu_2 - \delta)}
\]

Note that \( \frac{d\dot{k}(0)}{dr^*} = \frac{dq(0)}{dr^*} \), because in this specification \( J(J) \) is quadratic and \( J'' = 1 \).

b. Initial Rate of Change
\[
\frac{dq(0)}{dr^*} = \frac{a_{24} (\mu_1 + \mu_2)}{\tilde{b} (\mu_1 + \mu_2 - \delta)}
\]

4. Shadow Value of Wealth

a. Initial Jump

\[
\frac{d\lambda(0)}{dr^*} = \frac{\delta}{\tilde{b} a_{41}} - \frac{\tilde{\lambda}}{\mu_2 \mu_1} \left[ \frac{\mu_1 \mu_2 + a_{23}}{\mu_1 + \mu_2 - \delta} \right]
\]

b. Initial Rate of Change

\[
\frac{d\dot{\lambda}(0)}{dr^*} = -\tilde{\lambda}
\]

5. Terms of Trade

a. Initial Jump

\[
\frac{d\sigma(0)}{dr^*} = \frac{1}{(\tilde{\sigma}^2 + \tilde{x})} \left[ \tilde{x}^2 \frac{d\lambda(0)}{dr^*} - \tilde{\sigma} \frac{dq(0)}{dr^*} \right]
\]

b. Initial Rate of Change

\[
\frac{d\dot{\sigma}(0)}{dr^*} = -\frac{1}{(\tilde{\sigma}^2 + \tilde{x})} \left[ \tilde{x}^2 \tilde{\lambda} + \tilde{\sigma} a_{24} \tilde{b} \right]
\]

6. Domestic Interest Rate
a. Initial Jump
\[
\frac{dr(0)}{dr^*} = 1 + \frac{d\sigma(0) / dr^*}{\bar{\sigma}}
\]

b. Initial Rate of Change
\[
\frac{\dot{r}(0)}{dr^*} = \frac{a_{24}}{b(\mu_1 + \mu_2 - \delta)} \left[ \left( \mu_1 \frac{\mu_2}{\mu_1 + \mu_2 - \delta} \right) + a_{23} \right] - \frac{a_{24}a_{42}}{b(\mu_1 + \mu_2 - \delta)}
\]

7. Trade Balance, (in Terms of the Domestic Good)

a. Initial Jump
\[
\frac{dTB(0)}{dr^*} = -\frac{\delta}{b} + \frac{\tilde{\kappa} a_{41}}{\mu_1 \mu_2} \left[ \frac{\mu_1 \mu_2 + a_{23}}{(\mu_1 + \mu_2 - \delta)} \right] - \frac{a_{24}a_{42}}{b(\mu_1 + \mu_2 - \delta)}
\]

b. Initial Rate of Change
\[
\frac{dT\dot{B}(0)}{dr^*} = \frac{\tilde{\kappa} a_{41}}{b(\mu_1 + \mu_2 - \delta)} \left[ a_{42}(\mu_1 + \mu_2) + a_{43} \right]
\]

The Effect of the Permanent World Interest Shock on Welfare
\[
\frac{dW}{dr^*} = 2\frac{\tilde{\kappa}}{b} \left[ 1 - \frac{\mu_1 \mu_2}{(\delta - \mu_1)(\delta - \mu_2)} \right] \left[ (1 - \varepsilon)\bar{x}^2 + \varepsilon(\bar{\sigma}^2 + \bar{x})\bar{y}^2 \right]
\]

The welfare results in Table 1 of the paper are based on evaluating this expression at the steady state values.